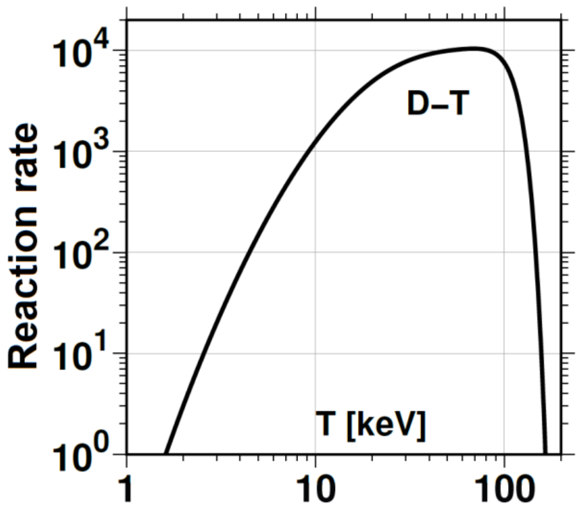
Simulation of a Fusion Power Plant

In this simulation task, we try to find the highest electrical power configuration of a tokamak-type nuclear fusion power plant, taking into account the operating constraints. In the reactors currently under development that implement controlled nuclear fusion, the two heavy isotopes of hydrogen, ^{2}H deuterium (D) and ^{3}H tritium (T), are used as fuel. If the temperature, density, and so-called cohesion of the fuel are in the appropriate range, the following fusion reaction can occur in large numbers:

$$\mathrm{D} + \mathrm{T} \longrightarrow \alpha + \mathrm{n} + 17{,}6\ \mathrm{MeV}.$$



The fusion power is proportional to how many D nuclei can potentially react with how many T nuclei. The behavior of fusion processes is better characterized by the volume density $n = N/V$, measured in units of units per cubic meter (m^{-3}). The total fusion power is therefore proportional ($\propto$) to the square of the density:

$$P_F \propto n^2 \cdot R(T).$$

The reaction rate $R(T)$ is a non-monotonic function of temperature (see figure), reaching its maximum at around 70 keV, after which the probability of fusion reactions decreases sharply. Significant fusion can only be expected when the fuel temperature reaches 10 − 20 keV, i.e. 100 − 200 million Kelvin. At such temperatures, all matter enters the so-called plasma state.

In this state, electrons are detached from the atomic nuclei and move independently of them. During the reaction, 80% of the energy is carried away by the neutron that is produced. Since it is an electrically neutral particle, it does not interact much with the electrons and atomic nuclei that make up the plasma, and therefore it leaves the plasma without significant energy loss and is absorbed in the cooling mantle. On the other hand, the α-particle, which carries 20% of the energy, is electrically charged. It transfers most of its energy to the plasma through continuous Coulomb interactions, and thus helps to heat and maintain the fusion plasma. But the reactor also needs external heating (P_K): firstly, to reach the operating temperature, secondly, to control the plasma, and thirdly (in certain cases) to achieve better performance.

The plasma that is constantly producing energy, has a high temperature, and therefore constantly tries to expand, must be held together in some way. In the Sun, this is ensured by gravity. Under terrestrial conditions, one option is to use magnetic fields. One such device is the tokamak, in which a torus-shaped plasma can be held together by high-power, superconducting electromagnetic coils. To hold the plasma together in a tokamak, also a current is required in the plasma, the magnetic field of which is added to the external field, creating magnetic confinement. The plasma current also provides a small amount of ohmic heating (P_Ω), but this only reaches a temperature of a few keV: as the temperature increases, the electrical resistance of the plasma decreases (!), so the efficiency of the ohmic heating also decreases.

In a fusion reactor, there must be a power balance in the steady state: the 0.2 P_F alpha heating and P_K external heating must just compensate the various radiation and transport losses in the plasma. The plasma temperature is determined by this balance for a given configuration. The main source of radiation loss in D−T P_S is the radiation of charged particles traveling along magnetic field lines, in continuous Coulomb interaction with other particles in the plasma, and being continuously accelerated by them. This depends in complex ways on the density, temperature, magnetic field, geometry, etc. In general, it can be said that radiation losses increase strongly with increasing magnetic field, density and temperature. The main driving force of transport losses is the turbulence caused by the high density and temperature inhomogeneity in the plasma (dense and hot inside, low density and cold outside), which also depends in a complex way on the plasma and power plant parameters.

The ratio of the thermal energy content of the plasma (W) to the fraction of the loss power P_T is the so-called confinement time ($\tau = W/P_T$) – the higher τ, the more efficient the reactor. In general, increasing the plasma current, magnetic field, and density slightly increases the confinement time. On the other hand, increasing the alpha and external heating power absorbed in the plasma decreases the confinement time.

In equilibrium, at constant power, the total heating power absorbed in the plasma should equal to the total power losses:

$$0{,}2 \cdot P_F + P_\Omega + \mathrm{P_K} = P_S + P_T.$$

The reactor operates at the highest temperature T where this stable power balance exists.

In the task, we want to maximize the electric power produced by the power plant. To calculate the electric power, it must be taken into account that the conversion of thermal power into electricity and the conversion of electric current into plasma heating also have their own efficiency: the former is taken as 35%, the latter is given as 1/3 for simplicity, so P_K heating requires $P_K/(1/3) = 3P_K$ electric power, which is how much less the power plant produces. The power required to maintain ohmic current drive and heating (P_Ω) should also be subtracted. Furthermore, the power plant also has a fixed electric consumption of 70 MW: for the operation of the circulation pumps, the superconducting magnet cooling systems, etc. Thus, the final electric power:

$$P_E = 0{,}35 \cdot (P_F + P_\Omega + P_K) - 3 \cdot P_K - P_\Omega - 70\ \text{MW}.$$

Operation limits

Although the tokamak is one of the "simplest" plasma confinement technologies, the presence of the plasma current in it causes several problems, which is why the available operating parameters are limited.

Current limit

In order for the plasma confinement to remain in stable equilibrium, the ratio of the plasma current I_p to the magnetic field B cannot exceed a certain limit. When this limit is exceeded, the plasma confinement collapses on a millisecond time scale, which is called plasma disruption. The simulation program was written to always keep the plasma current at the highest possible, but still safe value available for the given magnetic field. So we do not have to deal with this limit. However, this is the reason why the plasma current will also change when the magnetic field is changed.

Density limit

In tokamaks, also due to the presence of the plasma current, the plasma density n cannot exceed a certain limit. When this limit is exceeded, disruption also occurs. This is contrary to the goal that the fusion power will be greater when the density gets higher. The density limit is an empirical relationship, and its exact theoretical explanation is still the subject of research. One of our tasks will be to determine how the maximum achievable density depends on the input parameters (P external heating, B magnetic field).

Pressure limit

It has been found by theoretical calculations and empirically that if the pressure exceeds a critical value, disruption also occurs in the plasma. This is contrary to the point that we need a high-temperature and high-density plasma for a large number of fusion reactions. The maximum achievable pressure is proportional to the product of the magnetic field and plasma current:

$$[p_\alpha + n \cdot k \cdot T]_{\max} \propto B \cdot I,$$

where p_α is the pressure of high-energy (not yet thermal) α-particles. The B magnetic field is an input parameter chosen by us, while the I plasma current is automatically set by the program taking into account the above current limit. The simulation interface indicates what % of the pressure limit we are at with given parameters. 100% means disruption, at which point the simulation stops and must be restarted after the parameters are set.

The description and the tasks continue on the next page!

Measurement tasks

The numbers in square brackets [x] indicate the maximum points that can be awarded for each sub-task for information purposes. The jury may apply slightly different points in light of the solutions received (e.g. if there is a sub-task for which no good solution was received).

1) [0] Please read this complete document carefully!

A) Rethink the task! (Σ5)

2) [2] Think about and describe what the advantages and disadvantages of increasing the magnetic field, density, and external heating could be, based on the information in this measurement description!
3) [2] What can be expected, will electrical performance be best near or far from the density limit or pressure limit? Why?
4) [1] Get acquainted with the program management! Observe how the input controls work and what data the user interface prints. Describe which of these will definitely be needed to perform the tasks!

B) Operation limits (Σ10)

5) [6] Determine the density limit and the plasma current for at least 2 different heating powers and 3 magnetic fields. Plot both (separately) as a function of the magnetic field! If we manage to reach the pressure limit instead of the density limit, we can try e.g. another (B, n_{max}) combination, or reduce the heating power. What other output value(s) might be worth recording now?
6) [4] How do the density limit and plasma current depend on external heating and magnetic field? Describe what you experienced and how the results obtained can help in optimization!

C) Optimization (Σ10)

7) [7] Using your previous experiences, try to find which combination achieves the best electrical performance. Write down the experiences and the thought process. The logic behind the work and its documentation are worth more points than finding the maximum itself!
8) [3] Prepare a report about your experiences! Points will be awarded for the general clarity and followability of the report.

How to use the program

Input values can be adjusted by dragging the sliders, using the arrows, or by typing the values directly. Text boxes only accept numbers as input. The program provides feedback on the main parameters of the plasma and the power plant. In case of disruption, the error message must be acknowledged and the simulation restarted after adjusting the input parameters.

Advices

a) If the simulation runs into disruption, note which limit you have reached.
b) It is not necessary (and it is even not possible without p_α) to calculate the pressure limit value. You cannot set the temperature T directly. Useful is to understand how it depends on B and P.
c) The best electrical performance is not equal to the highest score!
d) The points are basically determined by the documentation of the work and the thought process. Instead of a try and error strategy, strive for logical work and describe your thought process: what you did and why.
e) The report can be submitted on paper, electronically, or a combination of both. The name of each file should clearly include your ID code. The paper report should clearly refer to the files.

(*Note*: the simulation is based on real tokamak physics and existing power plant designs. However, for simplicity of implementation, certain complex physical effects and technical limitations are intentionally not modeled, so the results may not always reflect the values expected in reality.)

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