Angular distribution of scattered particles in the simulation of the Rutherford experiment

The **scattering experiments** deliver the most information about the properties of microscopic particles. One of the first scattering experiments was made by **Lord Ernest Rutherford**, which led finally to the discovery of the atomic nucleus. With this program we simulate this experiment and clarify its interpretation.

At the beginning of the 20th century the **Thomson-model** of the atom was accepted. In this model the atom consisted of a heavy, positively charged „pudding", which had the size of the atom, and small, light, negatively charged particles – the electrons – were sitting inside, making the whole electrically neutral. Rutherford studied the scattering of high energy alpha particles on a thin gold foil. Since he knew that the positively alpha particles were much heavier than the electrons, therefore only the heavy part of the atom – the positively charged „pudding" - could make them deviate. He supposed that the only interaction between the two particles is the electrostatic Coulomb-interaction (repulsion), and this will determine the scattering of the particles.

He discovered that a few alpha-particle nearly „bounced back" from the gold foil, i.e. they scattered at large angles. From this he concluded that that the height of the repulsive potential hill must be higher than the kinetic energy of the alpha-particle, i.e. $E_{alpha} < E_{pot,\max}$.

As we have shown[1] $E_{pot,\max} = \frac{3}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze \cdot 2e}{R}$, where Z is the atomic number of the gold (Z =79), e is the elementary charge and ε_0 is the vacuum permittivity. The quantity R in the formula is the radius of the homogeneously charge target sphere, which is to be determined.

From the experimental fact of the „bouncing back" follows that $E_{alpha} < E_{pot,\max} = \frac{3}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze \cdot 2e}{R}$,

and from this we get an upper limit for the unknown R radius: $R < \frac{3}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze \cdot 2e}{E_{alpha}}$. Every quantity is known on the right hand side (Z = 79, E_{alpha} = 4,8 MeV), so, after substituting them into the formula we get $R < 7{,}11 \cdot 10^{-14}\,\mathrm{m}$. This is about ten thousand times smaller than the radius of the atom in the Thomson model. This way we arrive to the discovery of the small but heavy, positively charged **atomic nucleus**, and get **an estimation (upper limit) for its radius**..

However, there is a problem in the previous logic. The whole derivation is based on the **assumption** that only Coulomb-interaction is working between the target and projectile particle. If, for example this is not true, and the two particles interact like two billiard balls (hard spheres), then the alpha particle can bounce back from a much larger atom as well, and we cannot conclude that the nucleus must be small!

How could it be determined **what kind of interaction is acting between the two particles (target and projectile)?**

Rutherford realized that the angular distribution of the scattered particles contains much more information than what the above simple inequality expresses. Let us denote the scattering angle by ϑ. If a large number of particles are started, obviously not all of them will be scattered in the same way; one will more, another will less deviate from its original direction (think about the billiard balls: not every collision is the same; one hits directly, another ball is just scraping the target). So, let us collect and sort the scattered particles according their scattering angle, and let us analyse the distribution!

Rutherford deduced mathematically that if **only the Coulomb interaction** is acting between the atom and the alpha particles (and, if the alpha particle does not „penetrate" into the Thomson atom), the angular distribution of the scattered particles is proportional to the following expression:

[1] Click on the „***Info***" between the „Force" and the „Angular distribution" in the Simulation window

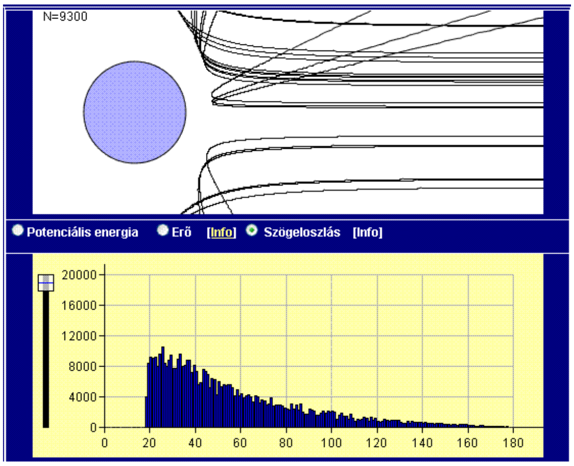
$$\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{(Ze)^2 \cdot (2e)^2}{16 \cdot E_{alpha}^2} \cdot \frac{1}{\sin^4\left(\frac{\vartheta}{2}\right)},$$

where the meaning of the different variables have already been explained earlier.

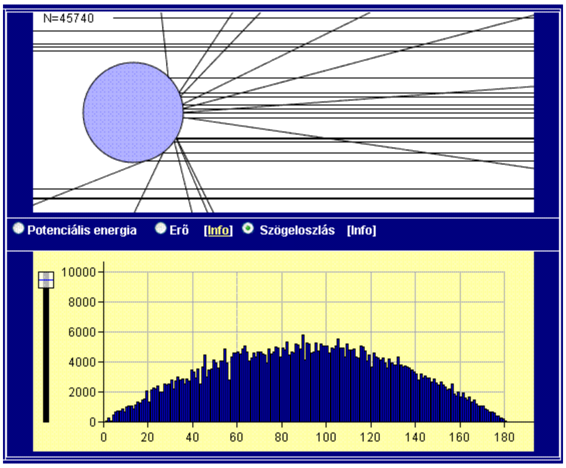
> ***Note:*** Now we already know, that in the world of the micro particles the *quantum mechanics* should be used. Rutherford could not be aware of that in 1911, since the quantum mechanics was developed only 15 years later! Therefore Rutherford could use only the *classical physics!* Today we know already, that the *Coulomb-interaction* is the **only one**, where the classical and quantum mechanical calculations give exactly the same results! Mother Nature was especially kind to Rutherford (and to the physicists) that allowed them to get good results even with a theory that in principle could not be used!

This simulation shows that the type of the interaction strongly influences the angular distribution of the scattered particles. We can choose between the „Rutherford-type” Coulomb-interaction, and the „hard sphere” interaction. The simulation gathers and sorts the scattered particles before our eyes. This way the angular distribution grows up directly in front of us, as if we counted the hits of an alpha detector array in a real experiment. It can easily be shown that the angular distribution of the scattered particles on a hard sphere is proportional to $\sin\vartheta$ (see later, in point 4. of the Remarks section).

The following figures show the angular distributions on „Thomson type” gold atoms with R = 50 fm radius, and also the trajectories of a few simulated alpha particles of 3 MeV energy.



Upper part: Simulation of Rutherford-scattering
Lower part: Angular distribution of Rutherford-scattering



Upper part: Simulation of hard sphere scattering
Lower part: Angular distribution of hard sphere scattering

On the upper parts of the figure the particles come from right to left, and approach the light-blue Thomson atom. On the left hand side one sees that the long range Coulomb interaction causes the trajectories to bend even far from the atom – the particles fly on hyperbolic trajectories. The „hard sphere” does not interact with the particles which are far away, but when they hit the atom, they bounce away like billiard balls.

> ***Remark:*** In the simulation the target is „fixed”, it does not get either energy or momentum from the scattered particles. In reality the target get kicked as well, although only a bit, since its mass is usually much higher than that of the alpha particle.

Although the single **trajectories** of the individual particles cannot be measured experimentally (at least not in the Rutherford experiment), the **angular distribution** of the scattered particles can be determined with detectors placed at different angles. It is obvious that the two angular distributions differ very much. The angular distribution shows a decreasing tendency at the Rutherford scattering (a $\frac{1}{\sin^4\frac{\vartheta}{2}}$ function would fit it well), whereas the $\sin\vartheta$ shape of the hard sphere distribution can also clearly seen.

Now you might better understand that the angular distributions provide a lot of information on the interaction of micro-particles for the nuclear and particle physicists!

Task: Using the simulation study how the angular distribution changes in the case of Coulomb scattering, if the alpha-particle can penetrate into the Thomson atom! For example look at the scattering of alpha particles with E = 6,7 MeV, and 6,85 MeV energies on a Thomson atom with a radius of R = 50 fm!

Remarks:

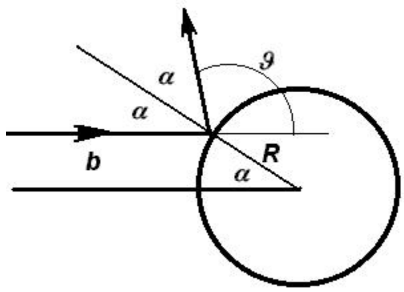
1) The range of the Coulomb-field is very large (infinity), therefore very small scattering angles would be caused only for particles flying really far away from the target. Since the simulation „space" is finite, therefore the particles which are scattered to small angles are „missing" from the plots.

2) The simulation draws *b*, the „impact parameter" (see in point 4. below, and in the figure below) statistically and uniformly along the vertical axis. However, the real space is 3D, which means that all points which lie on a circle with $2\pi \cdot b$ circumference have *b* impact parameter. Because of the cylindrical symmetry every particle that starts from any point of this circle will be deviated by the same angle. Therefore the „weight" of a particle starting with a *b* impact parameter will be proportional to *b*. The simulation takes this into account; therefore the total number of the particles in the angular distribution (the integral of the angular distribution function) is more than the number of the started particles.

3) The angular distribution for the hard sphere will be constructed only by the particles that were actually scattered (to avoid the many zero-angle scattering). Of course the previously mentioned "weight" is taken into account. Those particles that do not hit the sphere (and their scattering angles are zero) are neglected. At the Rutherford-scattering every particle will be deviated somehow (because of the long range of the Coulomb interaction), therefore every particle contributes to the angular distribution.

4) Deduction of the angular distribution of a hard sphere:

Consider the scattering shown in the picture. A particle with b impact parameter leaves at ϑ scattering angle. Obviously $\vartheta = \pi - 2\alpha$, and $b = R \cdot \sin\alpha$. For decreasing impact parameter $(b - \mathrm{d}b)$, the scattering angle increases: $(\vartheta + \mathrm{d}\vartheta)$. Complete bounce back $(\vartheta = \pi)$ occurs at $b = 0$ impact parameter. Let us suppose, that N particles are scattered in total. Let us calculate the number of particles $\mathrm{d}N$, which are scattered into the $(\vartheta + \mathrm{d}\vartheta)$ angular range!



The fraction of the particles that arrive in the circular ring $(b, b - \mathrm{d}b)$ interval, will be scattered into the $(\vartheta + \mathrm{d}\vartheta)$ angular range:

$$\mathrm{d}N = N \cdot \frac{2\pi b \cdot |\mathrm{d}b|}{\pi \cdot R^2} \qquad (*)$$

Now we have to switch over to ϑ and $\mathrm{d}\vartheta$ instead of $b \cdot |\mathrm{d}b|$. Because of the geometry $\alpha = \frac{\pi}{2} - \frac{\vartheta}{2}$, therefore $b = R \cdot \sin\alpha = R \cdot \sin\left(\frac{\pi}{2} - \frac{\vartheta}{2}\right) = R \cdot \cos\frac{\vartheta}{2}$. Differentiating it: $\mathrm{d}b = -\frac{R}{2}\sin\frac{\vartheta}{2} \cdot \mathrm{d}\vartheta$.

Using all these: $b \cdot |\mathrm{d}b| = R \cdot \cos\frac{\vartheta}{2} \cdot \frac{R}{2} \cdot \sin\frac{\vartheta}{2} \cdot \mathrm{d}\vartheta = \frac{R^2}{4} \cdot \left(2\sin\frac{\vartheta}{2} \cdot \cos\frac{\vartheta}{2}\right) \cdot \mathrm{d}\vartheta = \frac{R^2}{4} \cdot \sin\vartheta \cdot \mathrm{d}\vartheta$.

Finally, substituting this into the (*) equation we get:

$$\frac{\mathrm{d}N}{\mathrm{d}\vartheta} = \frac{N}{\pi \cdot R^2} \cdot 2\pi \cdot \frac{R^2}{4} \cdot \sin\vartheta = \frac{N}{2} \cdot \sin\vartheta.$$

Summarizing: $\boxed{\frac{\mathrm{d}N}{\mathrm{d}\vartheta} = \frac{N}{2} \cdot \sin\vartheta}$.

This shows that the angular distribution is proportional to the sine of the scattering angle.

As a check let us add (integrate) the number of scattered particles for all possible scattering angles, i.e. for the $(0, \pi)$ interval:

$\int_0^{\pi} \left(\frac{\mathrm{d}N}{\mathrm{d}\vartheta} \right) \mathrm{d}\vartheta = \frac{N}{2} \int_0^{\pi} \sin\vartheta \cdot \mathrm{d}\vartheta = N$, as it should be.