National Leo Szilárd Physics Competition 2021.
Computer simulation task
Theoretical Introduction and Description of Tasks

Modelling Positron Annihilation Tomography (PET) diagnostics

Before going further it is noted that there are several significant differences between a real PET scan (or PET device) and the current simulation model (see a photo of a real PET device).

- A real PET scan examines the body in three dimensions (3D), the current simulation is only two-dimensional.
- The real PET device has a large number of detectors in the ring(s) surrounding the patient, which therefore do not need to be rotated around the patient. In our simulation we use only one pair of detectors - "opposite" to each other.
- In order to simulate many detectors with this detector pair, the relative position of the patient and the detector pair must be changed. To do this, the table holding the patient can be rotated. (It is not necessary to rotate the table of a real PET machine).
- In a real PET device, the table holding the patient can be pushed into the detector ring, here the pair of detectors can be moved along one of the dimensions.

The task

The patient is given a positron emitting radioactive isotope (usually ^{18}F) intravenously. The radioactive isotope is attached to a sugar molecule and therefore gets enriched in cancerous tumor cells because of its increased metabolism. If one has a cancerous tumor in more than one place in the body, the radioactive isotope bound to the sugar molecule will be enriched in all those places. **The task is to find these cancerous tumors!**

The operation principle

The positron is an electrically charged particle, therefore it is slowed down very rapidly in body tissues, and encounters an electron near the emission site (less than 1 mm away) with which it annihilates. In this reaction two gamma photons with an energy of 511 keV each are generated, which are emitted in opposite directions (180°).

> For the sake of accuracy, we note that the angle is exactly 180° only in the center of mass system of the electron-positron pair. In the laboratory system the exact angle of the emission depends on the velocity of the positron-electron pair at the moment of annihilation. This also makes it possible to map the velocity distribution of electrons in matter by measuring the angle of the two gamma photons very accurately. Therefore, the positron annihilation technique is also used by solid state physicists. However, the angular deviation from 180° is so small that it can be completely neglected for PET applications, and the angle of the emitted gammas can be considered to be exactly 180°.

These gamma photons can be scattered or even be absorbed in the body along their path, but they are also likely to leave the body without interaction. We are interested in the latter photons, because if we detect them (both at the same time, in so-called coincidence), we can determine a direction along which they are emitted. If we can identify several such directions - "crossing" each-other - then the place of the emission can be determined. (In a real PET device, these different directions can be determined using detectors that count simultaneously in the detector ring. Therefore, in the simulation, it is necessary to rotate the table containing the patient.)

Some measurement problems to consider!

- *Angular resolution*. A line is defined by two points. Therefore, in order to determine the exact emission line, the exact location of the detection must be determined. However, all real detectors are of finite size. Therefore, the location of the detection is never point-like, i.e. the line of the emission cannot be determined very accurately. The smaller the size of a detector (at least the surface of the detector pointing towards the emission point), the more accurately we can determine the emission line. In the simulation, we can change this surface of the detectors with the help of radiation shielding slits. Obviously, the smaller the sensitive surface of the detector, the more accurate line-determination is possible.

- *Detection efficiency*. A gamma photon emitted from the source starts in a random direction (the other in the opposite direction). The smaller the surface of the detector facing the source, the less likely it is that gammas flying in random direction will just "hit" it. It can be seen from the previous point that the surface area of the detectors should be **decreased** in order to improve the angular resolution, and the surface area of the detectors should be **increased** in order to increase the detection efficiency. An appropriate compromise must be met between the two.

- *Real and random coincidences*. Events where two detectors simultaneously detect two gamma photons from **the same** annihilation event are called **true** coincidences. However, it is also conceivable that two gamma photons from **different** events arrive at the two detectors accidentally at the same time. These are **random** coincidences. It is not possible to tell whether a particular coincidence event is a real or a random event, however, there are ways to estimate the number of random coincidences. The number of random coincidences (N_V) can be determined by the following formula:

$$\frac{N_V}{T} = \frac{N_1}{T} \cdot \frac{N_2}{T} \cdot \Delta t \ ,$$

where N_1 and N_2 are the number of counts of the two detectors during the measuring time T, the Δt is the "coincidence time window" (resolution time), and N_V is the number of random coincidences also during the T measurement time[1]. (It is not necessary to derive and understand the above formula to complete the tasks, however, anyone who is interested can see it in the Appendix.)

The formula provides also guidance on how to reduce the number of random coincidences: on the one hand, the "coincidence window" should be kept to a minimum (the time within which the system considers "simultaneous" two signals arriving. Sometimes this is called time-resolution of the coincidence circuit). On the other hand, efforts should be made to reduce the detector's counting rate. This can be achieved by shielding the detectors well. (The "coincidence window" cannot be changed in the simulation, its value will have to be determined during one of the tasks.)

Tasks of the simulation round of the 2021 Leo Szilárd Physics Competition

1) *Task*: Get acquainted with the program! (See the separate Users' Guide!) (0 points)
2) *Task:* With optionally chosen settings, examine the effect of the aperture of the slits on the number of counts! Try to determine how the number of counts varies with the aperture of the slit! (3 points)
3) *Task:* Set the table to 90 degrees and set the slit size to maximum. Scan the patient with the *Autoscan#1* button. Then adjust the size of the slits to 2.2 cm and scan the patient

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[1] The meaning of the quantities in form N/T is the following: number of counts per unit time (intensity). If we introduce the notation $n = N/T$, then the above formula can also be written as: $n_V = n_1 \cdot n_2 \cdot \Delta t$. This form is often found in the literature.

again with the *Autoscan#2* button (do not change the position of the table). Compare the two measurement data sets in the *Graphs* table. Write down your experiences and the resulting conclusions in the minutes! (5 points)

4) *Task:* Based on the above experience, set the slits' apertures you judge the most appropriate, and set also the *Timing* value. Set the "guide-lines" at the various angle settings **and determine the positions of the hidden tumours** in the patient. (10 points)

5) *Task:* Determine the coincidence window (resolution time) of the coincidence. Remember that a measurement data also needs to determine its uncertainty (measurement error)! (5 points)

   (*Suggestion*: Choose situations where you are sure that there can be no true coincidences and use the N_V, N_1, N_2 and T values measured to determine the resolution time using the above formula). Also, choose the *Timing* value reasonably.

6) *Task:* When you think you are ready, don't forget to save your results using the *Data Backup* main menu item!! In this case, the program saves the current configuration, which is considered final, in a folder named after your code (this is a subfolder of the folder that also contains the program). This will allow the jury to restore your final configuration, which will also be examined when scoring, so this step is mandatory. If you also want to provide the jury with additional information (e.g. about an intermediate state of the solution), it is possible to save a screenshot (this is optional). You can do this by right-clicking on the graphs. In this case, the image of the diagnostic table as well as the image of the three graphs are saved. ATTENTION! Both the backup and screenshot file names are generated from your ID-code (so that they can be identified). Therefore, it is only possible to save a single data backup and a single screenshot file. When saving again, the previously saved file will be overwritten! (This point does not apply to the version available on the Internet. Data backup works differently there! Consult the Users' Manual.)

7) *Task:* Prepare a report on the measurement! (2 points for order, readability, etc.) The report can be prepared on paper or electronically, using one of the programs on the computer (e.g. Word, Excel, Notepad, etc.). If the report is written on paper, take a picture of it and upload it to the appropriate place in the "Classroom". Be sure to copy the entire contents of the directory created by the program, containing your result files, named from your code, to a pen-drive. If you have created the report (and/or other files) electronically, add them to the pen-drive as well (the file name clearly identifies your code), transfer it to the teacher's computer, and upload the entire material from there to the "Classroom" in the right place.

APPENDIX

Theoretical derivation of the random coincidence rate

The "coincidence" of electrical signals can be detected by means of an electronic AND circuit. It is a digital circuit that has at least two (or more) inputs and a single output. A signal is created at the output only if all inputs have a signal at the same time. In our case, we are looking for a coincidence of two signals, so it is a two-input AND circuit. Denote the two inputs with *Input#1* and *Input#2*! The length of the "time window" of the coincidence is denoted by Δt. This means that the circuit considers the two signals to arrive "simultaneously", if the time between them is shorter than Δt.

The input signal sequence of each of the detectors is assumed to be random and completely independent of each other (there are no "true" coincidences).

Determine now how many random coincidences are expected during time T if input#1 received N_1 counts, and input#2 received N_2. Suppose that $T \gg \Delta t$ (we measure for a very long time relative to the length of the coincidence time window, i.e. many signals arrive during the measuring time).

Consider the *Input#1* signal sequence. Each signal "opens" a Δt length "window" in the coincidence circuit within which, if a signal is received at *Input#2*, a coincidence signal is obtained at the output. During T time the circuit will be "open" for $N_1 \cdot \Delta t$ time. (Obviously $N_1 \cdot \Delta t < T$, otherwise *Input#1* would be constantly "open" and a signal coming into *Input#2* at any time would cause coincidence.) Since signals come randomly to *Input#2*, the probability that a **single** signal will accidentally arrive at such an open moment equals the quotient of the two time-periods: $\dfrac{N_1 \cdot \Delta t}{T}$. However, since *input#2* receives N_2 signals during T time, the expected number of coincidences is: $N_V = N_2 \cdot \dfrac{N_1 \cdot \Delta t}{T}$. Divide both sides by T and we arrive to the above formula: $\dfrac{N_V}{T} = \dfrac{N_1}{T} \cdot \dfrac{N_2}{T} \cdot \Delta t$.

It is immediately apparent from the formula that the roles of the two inputs (N_1 and N_2) are completely symmetric, so we would have obtained the same result even if we had not started from the signal sequence of *Input#1*, but from the signal sequence of *Input#2*.

The above reasoning is valid when we are sure that the two signal sequences are not correlated (random). It is also worth thinking about what happens when there are real coincidences as well. However, it is up to the reader to continue this thought.