Nuclear Physics

Practice 8

Exercise 1: Recoil energy in gamma-decay

Following the decay of ^{198}Au, three γ's are observed to be emitted from states in ^{198}Hg; their energies (in keV) are 1, 411.80441±0.00015; 2, 675.88743±0.00069 and 3, 1087.69033±0.00074. It is suggested that there are two excited states E_1 and E_2 in ^{198}Hg that are populated in the decay, and that the γ's correspond respectively to the transitions $E_1 \rightarrow E_0$, $E_2 \rightarrow E_1$, $E_2 \rightarrow E_0$ (where E_0 represents the ground state). If this hypothesis were correct, we would expect $E_{\gamma,1}+E_{\gamma,2}=E_{\gamma,3}$, which is almost but not quite true according to the experimental uncertainties. Show how the proper inclusion of the nuclear recoil resolves the discrepancy.

Solution:

The three γ-decay energies are the following:

$$E_{\gamma,1} = 411.80441 \pm 0.00015 \text{ keV}$$

$$E_{\gamma,2} = 675.88743 \pm 0.00069 \text{ keV}$$

$$E_{\gamma,3} = 1087.69033 \pm 0.00074 \text{ keV}$$

Let us calculate the sum of the first two γ-energies:

$$E^{'}_{\gamma,3} = E_{\gamma,1} + E_{\gamma,2} = 1087.69184 \pm 0.00071 \text{ keV}$$

where the uncertainty of the energy was determined with the Gaussian error propagation formula:

$$\Delta E^{'}_{\gamma,3} = \sqrt{(\Delta E_{\gamma,1})^2 + (\Delta E_{\gamma,2})^2} = 0.00071 \text{ keV}$$

The difference between the two energies:

$$E^{'}_{\gamma,3} - E_{\gamma,3} = 0.00151 \pm 0.00103 \text{ keV}$$

Although the difference is small, the question arises whether there is some effect which has been yet neglected in the calculation. → Recoil of the nucleus!

The momentum conservation law:

$$\underline{p}_N + \underline{p}_\gamma = 0$$

the momentum of the γ-photon is expressed as the following:

$$p_\gamma = \frac{E_\gamma}{c}$$

substituting this back to the energy conservation law, the recoil energy will be:

$$E_N = \frac{p_N^2}{2m_N} = \frac{E_\gamma^2}{2m_N c^2}$$

The corrected γ-energies (the uncertainties also changed, but the change is smaller than the last digit):

$$\tilde{E}_{\gamma,1} = 411.80487 \pm 0.00015 \text{ keV}$$

$$\tilde{E}_{\gamma,2} = 675.88867 \pm 0.00069 \text{ keV}$$

$$\tilde{E}_{\gamma,3} = 1087.69354 \pm 0.00074 \text{ keV}$$

Let us calculate now the sum of the first two corrected γ-energies:

$$\tilde{E}'_{\gamma,3} = \tilde{E}_{\gamma,1} + \tilde{E}_{\gamma,2} = 1087.69354 \pm 0.00071 \text{ keV}$$

where the uncertainty of the energy was determined again with the Gaussian error propagation formula:

$$\Delta\tilde{E}'_{\gamma,3} = \sqrt{(\Delta\tilde{E}_{\gamma,1})^2 + (\Delta\tilde{E}_{\gamma,2})^2} = 0.00071 \text{ keV}$$

The difference between the two corrected energies:

$$\tilde{E}'_{\gamma,3} - \tilde{E}_{\gamma,3} = 0 \pm 0.00103 \text{ keV}$$

Exercise 2: Resonance absorption of γ-radiation and the Mössbauer effect

The phenomenon of resonance absorption has been observed long time ago in atomic spectroscopy. However, resonance absorption of γ-radiation in nuclei could not be observed experimentally because of the effect of nuclear recoil is much greater than in atomic spectroscopy. The reason behind this can be understood through investigating the absorption cross-section around the resonance (this formula applies only for absorption in the nucleus, i.e. without reactions with the atomic electrons):

$$\sigma(E_\gamma) = \sigma_0 \frac{(\Gamma/2)^2}{\left[E_\gamma - (\Delta E + E_R)\right]^2 + (\Gamma/2)^2}$$

where σ_0 is the absorption cross-section at the exact resonant energy, E_γ is the energy of the γ-photon, ΔE is the energy difference between the initial and the final state. E_R is the recoil energy, which we have already calculated in the previous example:

$$E_R = \frac{p_\gamma^2}{2M} = \frac{E_\gamma^2}{2Mc^2}$$

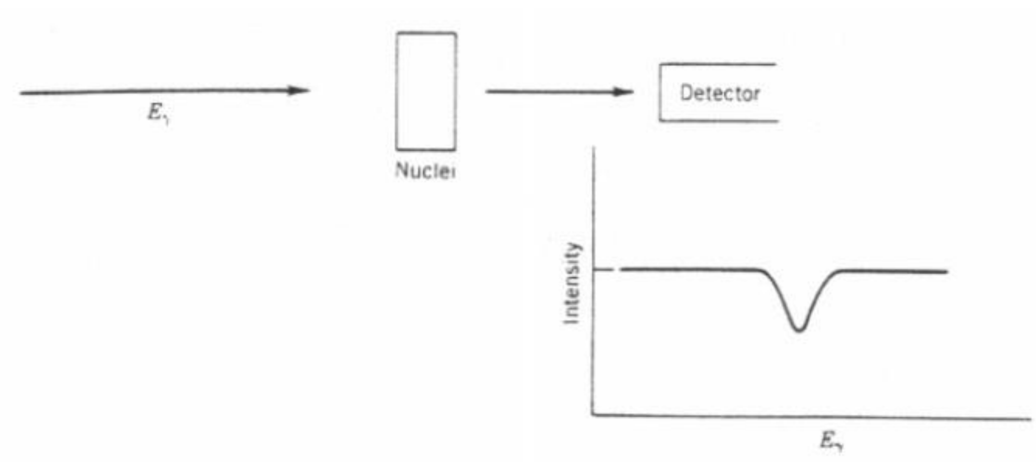
For example let us consider the γ-decay of ^{198}Hg with E_γ=412 keV energy:

$$E_R = 0.46 \text{ eV}$$

Γ is the natural linewidth of the initial state. For any given state with a mean lifetime τ, the measurement of the energy of the state gives a distribution with the corresponding linewidth:

$$\Gamma = \frac{\hbar}{\tau}$$

The schematic view of a resonance absorption experiment would look like the this:



In practice we would be unlikely to observe the natural linewidth Γ. A primary additional contribution is the Doppler-broadening due to the Maxwellian-distribution of the velocities, Δ:

$$\Delta = 2\sqrt{\ln(2)}E_\gamma\sqrt{\frac{2kT}{Mc^2}}$$

The Doppler-shifted energies:

$$E'_\gamma = E_\gamma\left(1 \pm \frac{v_x}{c}\right)$$

and the Maxwellian-distribution is proportional to:

$$f(v_x, v_y, v_z) \sim \exp\left(-\frac{\frac{1}{2}Mv_x^2}{kT}\right)$$

The Gaussian energy distribution will be therefore:

$$f(E'_\gamma) \sim \exp\left(-\frac{Mc^2}{2kT}\left(1 - \frac{E'_\gamma}{E_\gamma}\right)^2\right)$$

from this the formula for the Doppler-broadening can be obtained by calculating the corresponding FWHM $(\mathrm{FWHM} = 2\sqrt{2\ln(2)}\sigma)$. At room temperature kT=0.025 eV, therefore for example for the ^{198}Hg 412 keV γ-transition, the Doppler-broadening is:

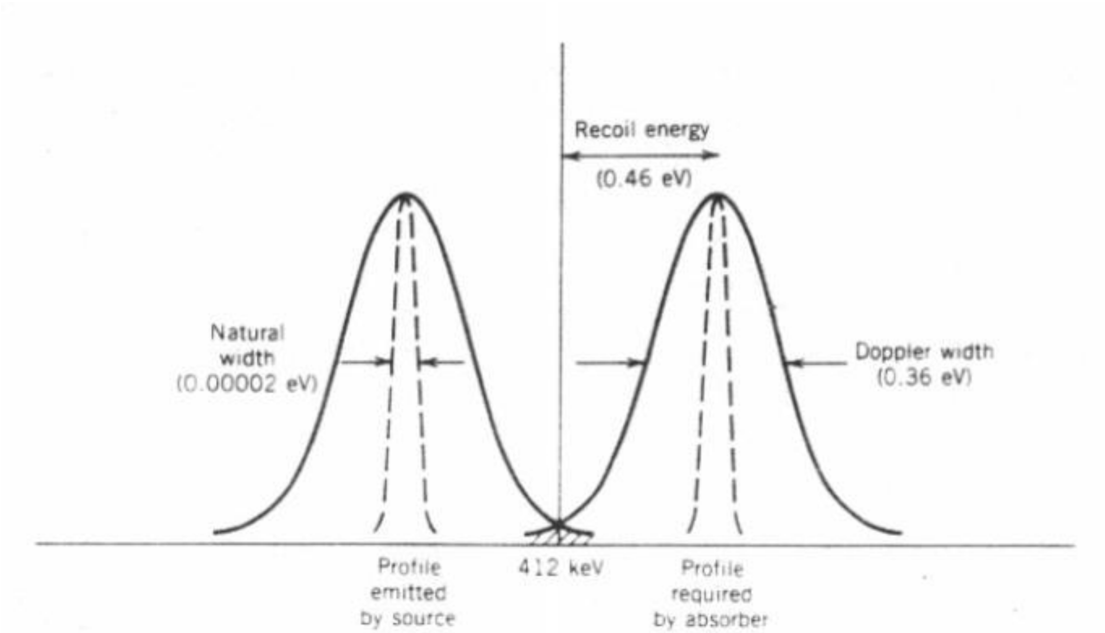
$$\Delta \approx 0.36\,\mathrm{eV}$$

which dominates the natural linewidth which is for the considered ^{198}Hg excited state $2\cdot10^{-5}$ eV. Even in thermal contact with 4 K liquid helium reservoir, the Doppler-broadening is:

$$\Delta \approx 0.042\ \mathrm{eV}$$

Tunable photons of the sort needed for the resonance experiment do not exist (the best one can do is brehmsstrahlung or synchrotron radiation). In the laboratory one has to work with discrete γ-energies. Therefore we must find a source which emits γ-radiation of an energy within at most 0.1 eV of the desired resonant energy $\Delta E+E_R$. This is very unlikely, especially with proper multipole characteristics. It makes sense to try to use a source in which the γ-radiation is emitted in the same downward transition that we are trying to excite upwards by resonance absorption.

However, due to nuclear recoil, the source emits γ-photons with $\sim\Delta E-E_R$ energy, while the absorber would need $\sim\Delta E+E_R$ γ-energies. In most cases the overlap is minimal between the two distributions, which gives very small absorption probabilities.



Several techniques exist for overcoming the energy difference $2E_R$ between the source and the absorber:

1) Doppler-broadening

2) Doppler-shift:

Let us move the source toward the absorber at high speed:

$$E'_\gamma = E_\gamma\left(1+\frac{v}{c}\right)$$

The changed energy should be $E_\gamma+2E_R$, from which the needed velocity can be calculated:

$$E_\gamma + 2E_R = E_\gamma\left(1+\frac{v}{c}\right)$$

$$v = c\left(\frac{E_\gamma + 2E_R}{E_\gamma} - 1\right) = c\left(1+\frac{2E_R}{E_\gamma} - 1\right) = c\,\frac{2E_R}{E_\gamma} \approx 700\,\frac{\text{m}}{\text{s}}$$

Experiments of this type are usually done by attaching the source to the tip of a rotor of a centrifuge spinning at 10^4-10^5 revolutions per minute.

3) Mössbauer-effect

The most successful and useful technique for defeating the recoil problem is the Mössbauer-effect. In 1958, Rudolf Mössbauer performed a resonance absorption experiment using a source of ^{191}Ir, where the emitting and absorbing nuclei were bound in a crystal lattice. Typical binding energies are 1-10 eV, so there is not enough recoil energy for the atom to leave its lattice site. A classical picture is that the mass in the recoil formula becomes the mass of the entire solid. In addition, a certain fraction of the atoms in a lattice are in the vibrational ground state of thermal motion, and therefore show very little Doppler-broadening.

Since both the absorber and the source have the natural width, in the case of ^{198}Hg the total linewidth is $2\cdot10^{-5}$ eV. To demonstrate the phenomenon, if we move the source towards the absorber with such a speed that the Doppler-shift will be greater than the natural linewidth, the resonance will be destroyed. For ^{198}Hg this velocity is:

$$E_\gamma + 2\Gamma = E_\gamma\left(1+\frac{v}{c}\right)$$

$$v = c\left(\frac{E_\gamma + 2\Gamma}{E_\gamma} - 1\right) = c\left(1+\frac{2\Gamma}{E_\gamma} - 1\right) = c\frac{2\Gamma}{E_\gamma} \approx 29\,\frac{\mathrm{mm}}{\mathrm{s}}$$

With the Mössbauer-effect, one can measure relative energies with extreme precision. If we modify the environment of the absorber or the source such that the initial and final states are shifted with a very small δE, we can measure this energy if it is in the same order as the natural linewidth. For example for the ^{198}Hg, we can measure ~10^{-5} eV energy difference for ~10^{5} eV γ-ray energy, or an effect of one part in 10^{10}.