Nuclear Physics

Practice 7

Exercise 1: Neutrino capture reactions

For neutrino capture reactions $\nu + {}^{A}X \rightarrow e^{-} + {}^{A}Y$ show that the Q value can be calculated as $[M({}^{A}X) - M({}^{A}Y)]c^2$ using atomic masses. Neglecting the small kinetic energy given to the final nucleus (to conserve momentum), the Q value is equal to (-1) times the minimum energy the neutrino must have to cause the capture reaction. Calculate the minimum neutrino energy necessary for capture by ^{37}Cl, ^{71}Ga and ^{115}In. In the Davis experiment ^{37}Cl was used to detect neutrinos from solar fusion, however ^{71}Ga and ^{115}In were also proposed as neutrino detectors. Comment on the use of these detectors to observe neutrinos from the basic fusion reaction $p + p \rightarrow d + e^{+} + \nu$ and the positive beta-decay of ^{8}Be.

$$M({}^{37}\text{Cl}) = 36.965897 \text{ u}$$
$$M({}^{37}\text{Ar}) = 36.966770 \text{ u}$$
$$M({}^{71}\text{Ga}) = 70.924690 \text{ u}$$
$$M({}^{71}\text{Ge}) = 70.924939 \text{ u}$$
$$M({}^{115}\text{In}) = 114.903860 \text{ u}$$
$$M({}^{115}\text{Sn}) = 114.903324 \text{ u}$$

Solution:

$$Q = \left[m_\nu + M({}^{A}_{Z}X) - \left(m({}^{A}_{Z+1}Y) + Z \cdot m_e + m_e\right)\right]c^2 = \left[M({}^{A}_{Z}X) - M({}^{A}_{Z+1}Y)\right]c^2$$

The minimal neutrino energies:

$$E_{\nu,\min} = -\left[M({}^{37}\text{Cl}) - M({}^{37}\text{Ar})\right]c^2 = 0.813 \text{ MeV}$$
$$E_{\nu,\min} = -\left[M({}^{71}\text{Ga}) - M({}^{71}\text{Ge})\right]c^2 = 0.232 \text{ MeV}$$
$$E_{\nu,\min} = -\left[M({}^{115}\text{In}) - M({}^{115}\text{Sn})\right]c^2 = -0.499 \text{ MeV}$$

More information about the Davis experiment can be found under this link.

Exercise 2: Confined alpha-particle

Use the uncertainty principle to estimate the minimum uncertainty of the speed and kinetic energy of an alpha-particle confined to the interior of a heavy nucleus.

$h = 6.626070 \cdot 10^{-34} \ \frac{\text{kgm}^2}{\text{s}}$

$r_0 = 1.25 \text{ fm}$

$m_\alpha = 6.64466 \cdot 10^{-27} \text{ kg}$

Solution:

From the uncertainty principle:

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$

Since we confine the alpha-particle to the interior of a heavy nucleus, the Δx distance will be the diameter of the nucleus. The radius can be calculated from the following equation:

$$R = r_0 \cdot A^{1/3}$$

therefore the diameter and the Δx distance will be:

$$\Delta x = 2R = 2r_0 \cdot A^{1/3}$$

The uncertainty of the momentum of the alpha-particle (x direction) can be obtained by rearranging the uncertainty principle:

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{4r_0 A^{1/3}}$$

The minimum uncertainty of the velocity and the kinetic energy can be easily calculated using the appropriate formulae:

$$\Delta v_{x,\min} = \frac{\Delta p_{x\min}}{m_\alpha} = \frac{\hbar}{4r_0 A^{1/3} m_\alpha}$$

$$\Delta E_{kin,\min} = \frac{\Delta p_{\min}^2}{2m_\alpha} = \frac{\Delta p_{x,\min}^2}{2m_\alpha} + \frac{\Delta p_{y,\min}^2}{2m_\alpha} + \frac{\Delta p_{z,\min}^2}{2m_\alpha} = \frac{3\hbar^2}{32 r_0^2 A^{2/3} m_\alpha}$$

Substituting *A*=200 (order of magnitude approximation) into the velocity and kinetic energy we get:

$$\Delta v_{\min} = \sqrt{\Delta v_{x,\min}^2 + \Delta v_{y,\min}^2 + \Delta v_{z,\min}^2} = 9.40 \cdot 10^5 \ \frac{m}{s}$$

$$E_{kin,\min} = 18.33 \text{ keV}$$

Exercise 3: Alpha-decay energy

Calculate the energy created in the alpa-decay of ^{238}U. How much kinetic energy will get the ^{234}Th from the recoil?

$M(^{238}\mathrm{U}) = 238.050788\ \mathrm{u}$

$M(^{234}\mathrm{Th}) = 234.043601\ \mathrm{u}$

$m_\alpha = 6.64466 \cdot 10^{-27}\ \mathrm{kg}$

$m_e = 9.109383 \cdot 10^{-31}\ \mathrm{kg}$

$1\ \mathrm{u} = 1.660539 \cdot 10^{-27}\ \mathrm{kg}$

Solution:

The Q value can be calculated with the following formula:

$$Q = \left[M(^{238}\mathrm{U}) - M(^{234}\mathrm{Th}) - m_\alpha - 2 \cdot m_e\right] \cdot c^2$$

which gives:

$$Q = 4.26990\ \mathrm{MeV}$$

The recoil energy is determined by the conservation laws:

$$0 = \underline{p}_{234\mathrm{Th}} + \underline{p}_\alpha$$

$$Q = E_{kin,234\mathrm{Th}} + E_{kin,\alpha} = \frac{p_{234\mathrm{Th}}^2}{2M(^{234}\mathrm{Th})} + \frac{p_\alpha^2}{2m_\alpha}$$

rearranging we can get the momentum of ^{234}Th:

$$p_{234\mathrm{Th}} = \sqrt{Q\left(\frac{2M(^{234}\mathrm{Th})m_\alpha}{m_\alpha + M(^{234}\mathrm{Th})}\right)}$$

therefore the kinetic energy from the recoil will be:

$$E_{kin,234Th} = \frac{Q\left(\frac{2M(^{234}\mathrm{Th})m_\alpha}{m_\alpha + M(^{234}\mathrm{Th})}\right)}{2M(^{234}\mathrm{Th})} = Q\left(\frac{m_\alpha}{m_\alpha + M(^{234}\mathrm{Th})}\right) = 71.8\ \mathrm{keV}$$