Nuclear Physics

Practice 5

Exercise 1: Weizsäcker-formula

Consider the following alpha decay of ^{232}Th:

$$^{232}_{90}\text{Th} \rightarrow ^{228}_{88}\text{Ra} \rightarrow^{\beta} \ ?$$

How many beta-decays will follow the alpha-decay?

Solution:

The number of necessary beta-decays can be determined by the principle of minimum energy. This means that we search for the nucleus with minimum energy per nucleon at constant mass number:

$$\left.\left(\frac{E}{A}\right)\right|_{Z=?} = \min .$$

We will utilize the Weizsäcker-formula:

$$B = b_V A - b_F A^{2/3} - b_C \frac{Z^2}{A^{1/3}} - b_A \frac{(A-2Z)^2}{A} + b_P \delta(Z,A) A^{-1/2}$$

where we will neglect the effect of the pairing energy. We examine the minimum energy per nucleon:

$$\frac{E}{A} = -\frac{B}{A} = -b_V + b_F A^{-1/3} + b_C \frac{Z^2}{A^{4/3}} + b_A \frac{(A-2Z)^2}{A^2}$$

let us derivate this expression:

$$\frac{\partial}{\partial Z}\left(\frac{E}{A}\right) = 2b_C \frac{Z}{A^{4/3}} - 4b_A \frac{(A-2Z)}{A^2} = 2b_C \frac{Z}{A^{4/3}} - 4b_A \frac{1}{A} + 8b_A \frac{Z}{A^2} = 0$$

$$\frac{4b_A}{A} = \left(\frac{2b_C}{A^{4/3}} + \frac{8b_A}{A^2}\right) Z$$

The coefficients in the equation are the following: $b_A = 3.80 \cdot 10^{-12}$ J, $b_C = 0.11 \cdot 10^{-12}$ J

Therefore the Z we are searching for:

$$Z = \frac{4b_A}{A(2b_C A^{-4/3} + 8b_A A^{-2})} \approx 89.8 \Rightarrow 90$$

which equals two beta-decays ($^{228}_{88}\text{Ra} \rightarrow^{\beta} \ ^{228}_{89}\text{Ac} \rightarrow^{\beta} \ ^{228}_{90}\text{Th} \rightarrow^{\alpha} \ldots$).

Exercise 2: Strength of the four fundamental interactions

The strength of the force exerted in an interaction is described with the coupling constants. These are dimensionless physical quantities independent of the units used, which means that they cannot be eliminated by any choice of system of units. Usually the Lagrangian or the Hamiltonian of a system describing an interaction can be separated into a kinetic part and an interaction part. The coupling constant determines the strength of the interaction part with respect to the kinetic part, or between two sectors of the interaction part.

There are four fundamental forces in the standard model:

- strong interaction
- weak interaction
- electromagnetic force
- gravitational force

2) In the lecture we have seen the coupling constant for the weak interaction:

$$G = g\frac{M_n^2 c}{\hbar^3} \approx 10^{-5}$$

where g is the interaction „strength" in the H_β interaction operator, M_n is the mass of the nucleons, c is the speed of light and $\hbar$ is the Planck-constant. The g interaction strength was used in the calculation of the interaction matrix element in Fermi's „Golden rule":

$$\lambda(E_f) = \frac{2\pi}{\hbar} |V_{i,f}|^2 \; \rho(E_f)$$

$$V_{i,f} = g\int \Psi_f^* \hat{H}_\beta \Psi_i d^3r = g\int \psi_f^* \varphi_\nu^* \varphi_e^* \hat{H}_\beta \psi_i d^3r$$

since the final state is a nucleus + electron + antineutrino.

3) The coupling constant for the electromagnetic force is denoted by α and it is called the fine structure constant because it also appears in the description of the fine structure of atomic spectral lines. The fine structure constant has multiple interpretations, in quantum electrodynamics α determines the strength of the interaction between electrons and photons. The value of α is the following:

$$\alpha = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{\hbar c} \approx \frac{1}{137}$$

This expression can also be interpreted as the ratio of two energies: the energy needed to overcome the Coulomb-repulsion between two electrons with d distance apart and the energy of a single photon of wavelength $\lambda = 2\pi d$:

$$\alpha = \frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{d}}{\frac{hc}{\lambda}} = \frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{d}}{\frac{hc}{2\pi d}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{\hbar c}$$

4) The gravitational coupling constant α_g characterizes the gravitational attraction between two elementary particles having nonzero mass. Analogous to the fine structure constant, α_g is defined in the following way if the masses of two protons are used:

$$\alpha_g = \frac{Gm_p m_p}{\hbar c} \approx 6\cdot 10^{-39}$$

Compared to the fine structure constant, the ratio of the strength of the two forces can be calculated:

$$\frac{\alpha_g}{\alpha} \approx 8\cdot 10^{-37}$$

This ratio can also be estimated with the following thinking: let us examine the ratio between the electrical and gravitational force of two protons being d distance apart:

$$\frac{\alpha_g}{\alpha} = \frac{F_G}{F_E} = \frac{G\frac{m_p m_p}{d^2}}{\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{d^2}} = \frac{Gm_p m_p}{\frac{1}{4\pi\varepsilon_0}\cdot e^2} \approx 8\cdot 10^{-37}$$

1) Lastly, the coupling constant of the strong force is derived from quantum chromodynamics (QCD). The data describing the interactions between nucleons is consistent with a coupling constant

$$\alpha_s \approx 1$$

however it drops off at small distances, and strong force diminishes inside the nucleons. This phenomenon is called asymptotic freedom because the quarks approach a state where they can move freely in the tiny volume of the hadron.

Exercise 3: Neutrino energies in solar fusion

The proton-proton chain is thought to be the dominant source of energy generation in the Sun, where the initial proton-proton reaction establishes the basic rate for all subsequent reactions (see Figure 1). The pep reaction is named after the proton, electron and proton reaction. The deuterons produced in these two reactions fuse with protons to form a light isotope of helium. At this point the proton-proton chain breaks into three branches.

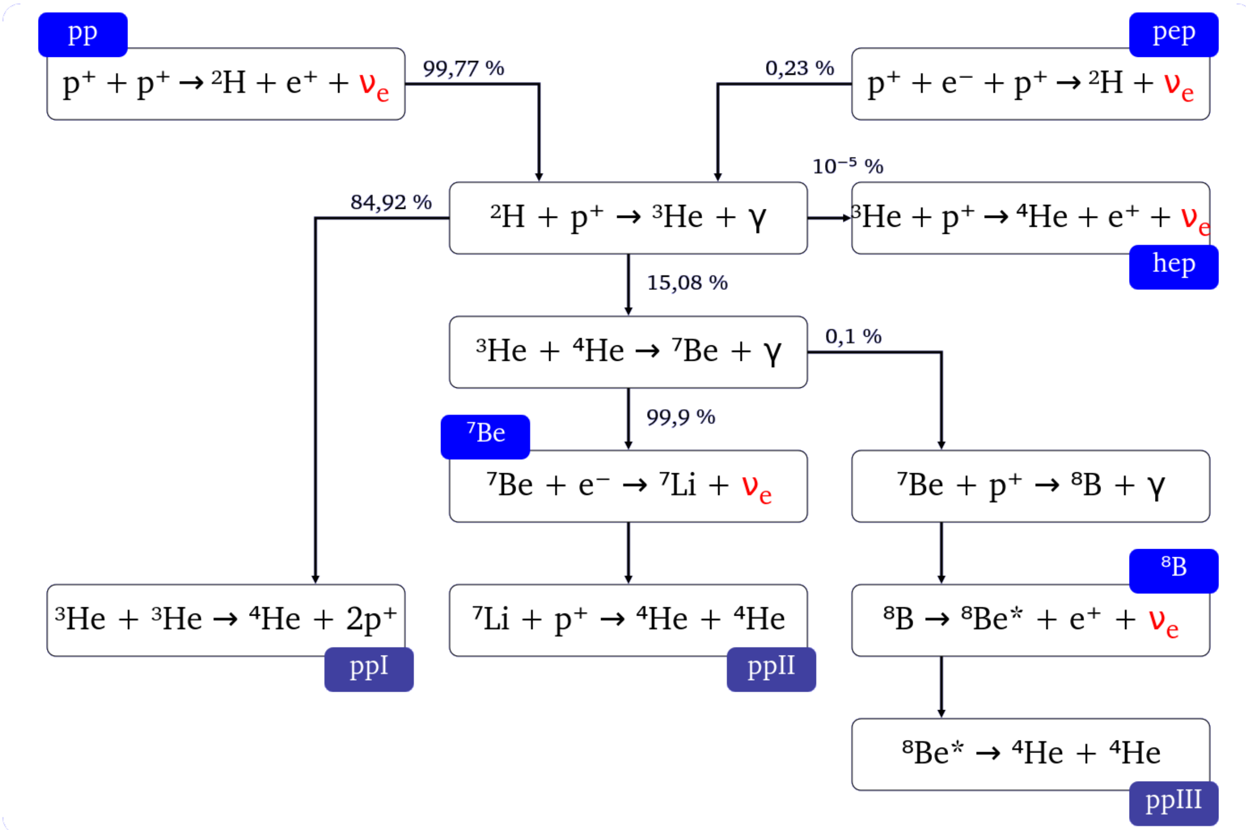

Figure 1: Proton–proton and electron-capture chain reactions in the Sun

a) Calculate the maximal energy of the neutrinos emitted in the solar fusion reaction $p+p \to d+e^+ + \nu$. Assume that the initial protons have negligible kinetic energies.

$$m_p c^2 = 938.272046 \text{ MeV}$$
$$m_d c^2 = 1875.612859 \text{ MeV}$$
$$m_e c^2 = 0.510999 \text{ MeV}$$

b) The most energetic neutrinos are emitted from the positive beta-decay of ^8B. Calculate the maximal energy of the emitted neutrinos, if the daughter ^8Be nucleus is created in an excited state of 2.84 MeV (see Figure 2). The masses of the ^8B and ^8Be atoms in atomic mass units are the following (1 u = 931.494095 MeV/c^2):

$$M(^8\text{B}) = 8.024606 \text{ u}$$

$M(^{8}\mathrm{Be}) = 8.005304\ \mathrm{u}$

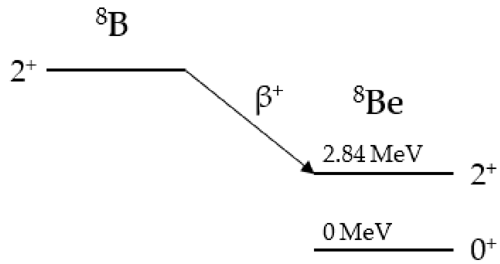

Figure 2: Decay scheme of the positive beta-decay of ^{8}B.

Solution:

a) Based on the masses of the initial two protons and the reaction products the Q value can be calculated as:

$$E_{\nu,\max} = Q = \left[2 \cdot m_p - (m_d + m_e + m_\nu)\right]c^2 \approx \left[2 \cdot m_p - m_d - m_e\right]c^2 = 0.420\ \mathrm{MeV}$$

where we neglected the mass of the neutrino.

b) The maximal energy of the emitted neutrinos equals the Q value in the β^+ decay minus the excitation energy:

$$E_{\nu,\max} = Q - 2.84\ \mathrm{MeV} = \left[M(^{8}B) - 5 \cdot m_e - M(^{8}Be) + 4 \cdot m_e - 2 \cdot m_e\right]c^2 - 2.84\ \mathrm{MeV} = 14.06\ \mathrm{MeV}$$