Nuclear Physics

Practice 4

Exercise 1: Natural radioactivity of granite

The average natural abundance of ^{238}U in granite is 4 ppm (ppm = 10^{-6}). Calculate the total activity of 1 tons of granite if the decay chain of ^{238}U ends as the following:

$$^{238}_{92}\text{U} \rightarrow \ldots \rightarrow ^{206}_{82}\text{Pb}$$

Hint: how many alpha- and beta-decays are present in the decay chain? The half-life of ^{238}U is

$$T_{1/2}^{238\text{U}} \cong 4.5 \cdot 10^9 \text{ years}$$

whereas the half-life of the daughter elements are orders of magnitudes lower. Consider the bulk of the granite as SiO_2!

Solution:

Since only the alpha-decay can decrease the mass number of the nucleus, and the mass number decreases by 32, eight alpha-decays are necessary to reach ^{206}Pb. The atomic number of the resulting isotope would be 92-16=76, therefore six β^--decays are also needed in the chain. The total number of radiactive decays is therefore 8+6=14.

Suppose that the granite mainly consists of SiO_2. The molar mass of SiO_2 is approximately:

$$M_{\text{SiO2}} = 28 + 2 \cdot 16 = 60 \ \frac{\text{g}}{\text{mol}}$$

From this we can calculate the amount of SiO_2 molecules in 1 tons of granite:

$$n_{\text{SiO2}} = \frac{m}{M_{\text{SiO2}}} = 16666.667 \text{ mol}$$

The amount of ^{238}U atoms is therefore:

$$n_{238\text{U}} = n_{\text{SiO2}} \cdot 4 \cdot 10^{-6} = 0.06667 \text{ mol}$$

which means that the number of ^{238}U atoms is (using the Avogadro-constant):

$$N_{238\text{U}} = n_{238\text{U}} \cdot N_A = 4.0147 \cdot 10^{22}$$

The activity of ^{238}U in the granite can be calculated by multiplying by the decay constant:

$$A_{238\text{U}} = \lambda_{238\text{U}} \cdot N_{238\text{U}} = \frac{\ln 2}{T_{1/2}^{238\text{U}}} \cdot N_{238\text{U}} = 0.196 \text{ MBq}$$

but this is not the total activity, since 14 decays are present per one ^{238}U nucleus in the decay chain in secular equilibrium. The total activity will be:

$$\sum A = 14 \cdot A_{238\text{U}} = 2.75 \text{ MBq}$$

Exercise 2: Radioactive dating

A wine expert finds two bottles of wine in a family wine cellar without any labels. He supposes that the method of preparation is the same in the two cases, but he cannot decide which wine is older. So he hires a physicist, who immediately knows that the solution will be given by the tritium (^{3}H) content of the wine, since it has 12.32 years half-life. He measures 1 cm^3 of each wine in a liquid scintillator. The total number of counts for the first sample was 320 during 20 hours, while for the second one he measured 340 during 30 hours. How much is the age difference between the two wines? Calculate the uncertainty of the result using the Gaussian uncertainty propagation formula!

Solution:

The measurement time is much smaller than the half-life of tritium therefore we can assume that the activity of the samples was constant during the measurement. This gives the following activities for the two samples:

$$A_1 = \frac{N_1}{t_1} = 4.444 \cdot 10^{-3} \text{ Bq}$$

$$A_2 = \frac{N_2}{t_2} = 3.148 \cdot 10^{-3} \text{ Bq}$$

so the second wine is older than the first one. The uncertainties of the N_1, N_2 counts are the standard deviations of the corresponding Poisson-distributions:

$$\Delta N_1 = \sqrt{N_1} = 17.889$$

$$\Delta N_2 = \sqrt{N_2} = 18.439$$

Therefore the uncertainty of the activities can be calculated by using the variance formula:

$$\Delta A_1 = \sqrt{\left(\frac{\partial A_1}{\partial N_1}\Delta N_1\right)^2} = \frac{\Delta N_1}{t_1} = 2.485 \cdot 10^{-4} \text{ Bq}$$

$$\Delta A_2 = \sqrt{\left(\frac{\partial A_2}{\partial N_2}\Delta N_2\right)^2} = \frac{\Delta N_2}{t_2} = 1.707 \cdot 10^{-4} \text{ Bq}$$

The time dependency of the activity is:

$$A_i(t) = A_{i,0} \exp(-\lambda t)$$

By the time of the measurement this gives:

$$A_1(t_m) = A_{1,0} \exp(-\lambda(t_m - \Delta t))$$

$$A_2(t_m) = A_{2,0} \exp(-\lambda t_m)$$

where Δt is the age difference between the two wines. $A_{1,0}=A_{2,0}$ because the method of preparation and the grape type was the same, therefore the ratio of the two activities:

$$\frac{A_2(t_m)}{A_1(t_m)} = \frac{\exp(-\lambda t_m)}{\exp(-\lambda(t_m - \Delta t))} = \exp(-\lambda \Delta t)$$

From this equation the age difference can be easily calculated:

$$\Delta t = -\frac{1}{\lambda} \ln \frac{A_2(t_m)}{A_1(t_m)} = 6.13 \pm 1.38 \text{ years}$$

where the uncertainty of the age difference was also calculated with the variance formula:

$$\Delta(\Delta t) = \sqrt{\left[\frac{\partial(\Delta t)}{\partial A_1}\Delta A_1\right]^2 + \left[\frac{\partial(\Delta t)}{\partial A_2}\Delta A_2\right]^2} = \sqrt{\left(\frac{\Delta A_1}{\lambda A_1}\right)^2 + \left(\frac{\Delta A_2}{\lambda A_2}\right)^2} = \frac{1}{\lambda}\sqrt{\frac{\Delta A_1^2}{A_1^2} + \frac{\Delta A_2^2}{A_2^2}}$$