

Nuclear Physics

Practice 2

Exercise 1: Cockcroft-Walton voltage multiplier

John D. Cockcroft and Ernest Walton at the Cavendish Laboratory in Cambridge, England, managed to build a voltage multiplier with which they could build up a potential of 800 kV from a transformer of 200 kV. They used the potential to accelerate protons down an evacuated tube eight feet long, and in 1932 when they put a lithium target at the end of the tube, found that the protons disintegrated the lithium nucleus into two alpha-particles.

In order to understand the working principle of the Cockcroft-Walton voltage multiplier, let us first consider a simple rectifier circuit (see on the left in Figure 1). The input voltage to this circuit is an alternating voltage of amplitude V_0 , which can be either a sinusoidal or square wave AC voltage. In the rectifier circuit only the positive half of the AC wave is passed to the point of measurement (A). In the case of a square wave, the voltage at point A will be $+V_0$ volts both in the positive half and in the negative half (see Figure 3). Now consider the mirrored circuit on the right in Figure 1, which is called the Villard doubler circuit. Comparing the two circuits, only the place of the ground (or reference) has changed, therefore the AC voltage will be added to the output voltage at A. This results in $V_0+V_0=2V_0$ volts output in the positive half and $-V_0+V_0=0$ volts in the negative half. So we have a doubled voltage output with respect that of the ordinary output voltage of a simple rectifier!

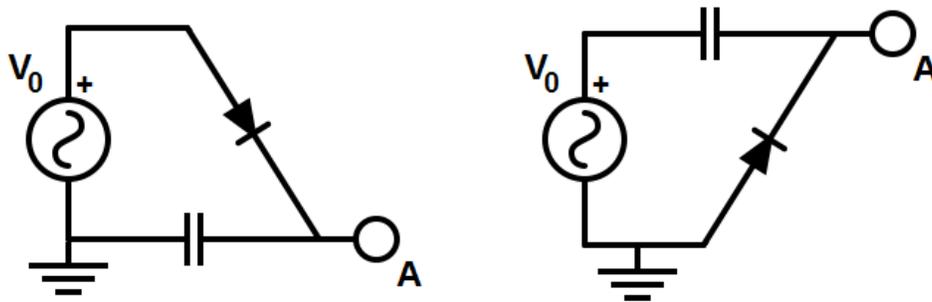


Figure 1: A rectifier circuit (left) and the Villard doubler circuit (right)

We can get constant DC output of $2V_0$ by extending the Villard doubler circuit to a Greinacher doubler circuit (see Figure 2). Recall that the output voltage of the Villard circuit is stepped going from a minimum of 0 volts to a maximum of $2V_0$ volts. This is the signal at the right hand side of the upper capacitor. During the time that this potential is at $2V_0$ volts, charge will flow through the new diode charging the lower capacitor to $2V_0$, so the lower capacitor will become charged to a permanent positive $2V_0$ DC volts with respect to ground, which is the left hand side of the lower capacitor. In the upper part of Figure 4 is the circuit element that will be added repeatedly to the Greinacher circuit in order to get the Cockcroft-Walton voltage multiplier.

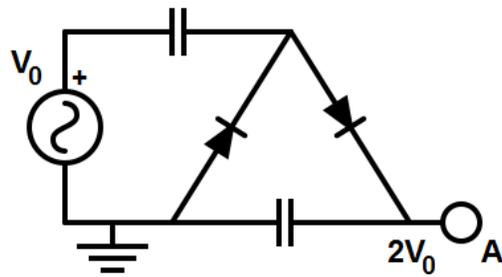


Figure 2: The Greinacher doubler circuit

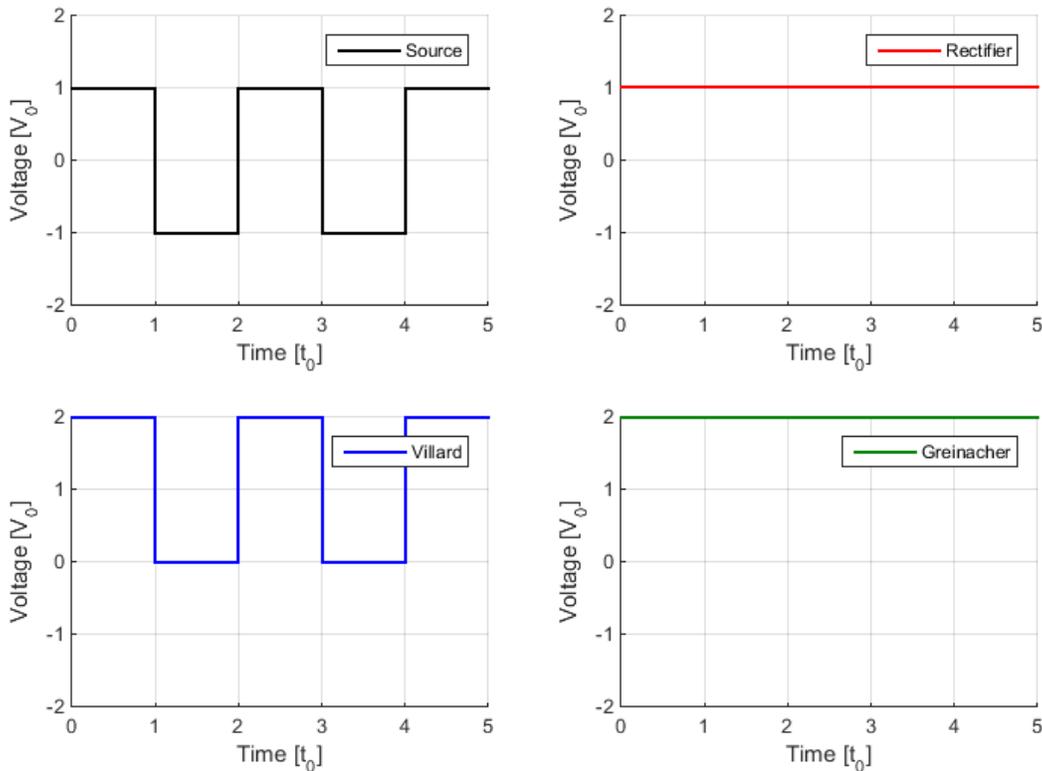


Figure 3: Measured voltages in the different circuits

The output of the Greinacher doubler at the top node which we call the upper voltage of the first stage is at an alternating stepped voltage of a maximum of $2V_0$ volts and a minimum of 0 volts. At the bottom node there is a constant DC voltage of $2V_0$. Considering only the repeatedly added elements with the upper AC and lower DC voltage a similar thinking brings us to the conclusion that for each stage of the Cockcroft-Walton voltage multiplier another $2V_0$ voltage is added to the bottom node with respect to the ground (or reference voltage), which was at the lower left part of the AC voltage.

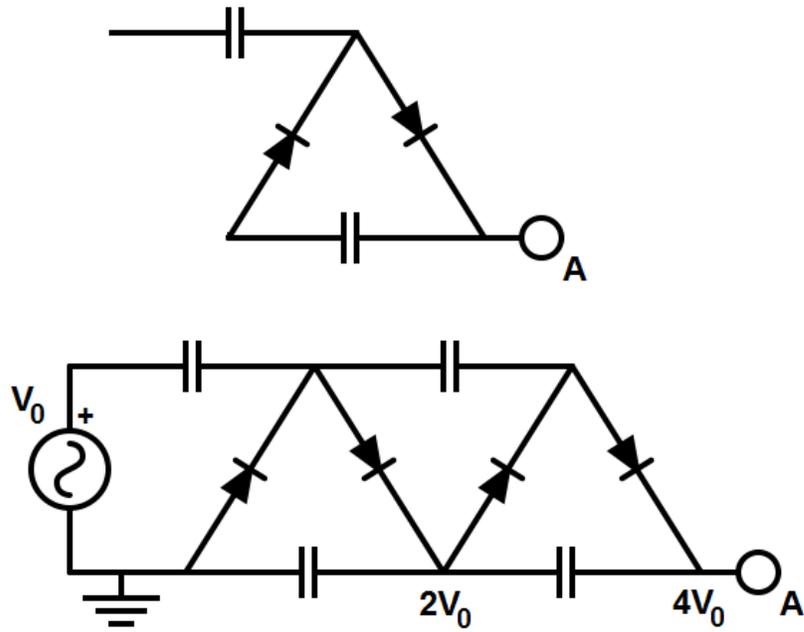


Figure 4: The Cockcroft-Walton voltage multiplier (bottom) and the repeated circuit element which has to be added to the Greinacher doubler circuit (top).

Exercise 2: Charged sphere potential (Thomson-atom)

Given a non-conductive charged sphere with homogeneous charge density:

$$\rho = \frac{Ze}{\frac{4R^3\pi}{3}}$$

where Z is the atomic number, e is the elementary charge and R is the radius of the Thomson-atom. The electric field can be determined by Gauss's law:

$$\oint_s \underline{E} d\mathbf{A} = \int_v \text{div} \underline{E} dV = \int_v \frac{\rho}{\epsilon_0} dV = \frac{1}{\epsilon_0} \int_v \rho dV$$

Because of the spherical symmetry and the isotropy E can be taken out from the integral:

$$\oint_s \underline{E} d\mathbf{A} = E \oint_s d\mathbf{A} = E \cdot 4r^2\pi$$

Two cases:

1) $r > R$

$$E \cdot 4r^2\pi = \frac{\rho}{\epsilon_0} \cdot \frac{4R^3\pi}{3} = \frac{1}{\epsilon_0} \cdot \frac{Ze}{\frac{4R^3\pi}{3}} \cdot \frac{4R^3\pi}{3} = \frac{1}{\epsilon_0} \cdot Ze$$

$$E = -\nabla\phi_1 = -\frac{\partial}{\partial r}\phi_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze}{r^2}$$

therefore the potential:

$$\phi_1 = -\int_{\infty}^r \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze}{r^2} dr = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze}{r}$$

2) $r \leq R$

In this case only part of the sphere is considered in the right hand side of the equation:

$$E \cdot 4r^2\pi = \frac{\rho}{\epsilon_0} \cdot \frac{4r^3\pi}{3} = \frac{1}{\epsilon_0} \cdot \frac{Ze}{\frac{4R^3\pi}{3}} \cdot \frac{4r^3\pi}{3} = \frac{1}{\epsilon_0} \cdot Ze \cdot \frac{r^3}{R^3}$$

$$E = -\nabla\phi_2 = -\frac{\partial}{\partial r}\phi_2 = \frac{1}{4\pi\epsilon_0} \cdot Ze \cdot \frac{r}{R^3}$$

therefore the potential (also fitting on the boundaries to ϕ_1):

$$\phi_2(r) = -\int_R^r \frac{1}{4\pi\epsilon_0} \cdot Ze \cdot \frac{r}{R^3} dr + \phi_1(R) = -\frac{1}{2} \cdot \frac{1}{4\pi\epsilon_0} \cdot Ze \cdot \left(\frac{r^2}{R^3} - \frac{R^2}{R^3} \right) + \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze}{R} = \%$$

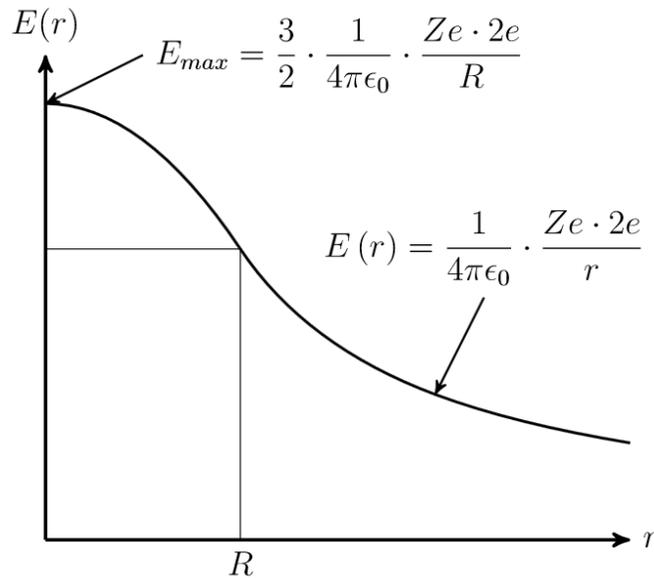
$$\% = \frac{1}{2} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze}{R^3} \cdot (3R^2 - r^2)$$

The resultant potential:

$$\phi(r) = \begin{cases} \frac{1}{2} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze}{R^3} \cdot (3R^2 - r^2), & \text{if } r \leq R \\ \frac{1}{4\pi\epsilon_0} \cdot Ze \cdot \frac{r}{R^3}, & \text{if } r > R \end{cases}$$

The kinetic energy required for an alpha-particle to reach r distance:

$$E_{kin,req}(r) = \phi(r) \cdot 2e$$



The kinetic energy required to pass through the charged sphere:

$$E_{kin,max} = E_{kin,req}(0) = \frac{3}{2} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze \cdot 2e}{R}$$