Nuclear Physics

Practice 1

Exercise 1: Dempster-type mass spectrometer

How far will be the two impact points of single ionizated ^{23}Na and ^{24}Mg ions on the screen after passing one semicircle?

The mass difference between the two ions is Δm=1.20333 u (u=$1.66053 \cdot 10^{-27}$ kg).

The fields of the velocity selector are E=1000 N/C and B_1=0.1 T, and the magnetic induction of the the diverting magnetic field is B_2 = 0.01 T.

The schematic structure of the mass spectrometer is the following:

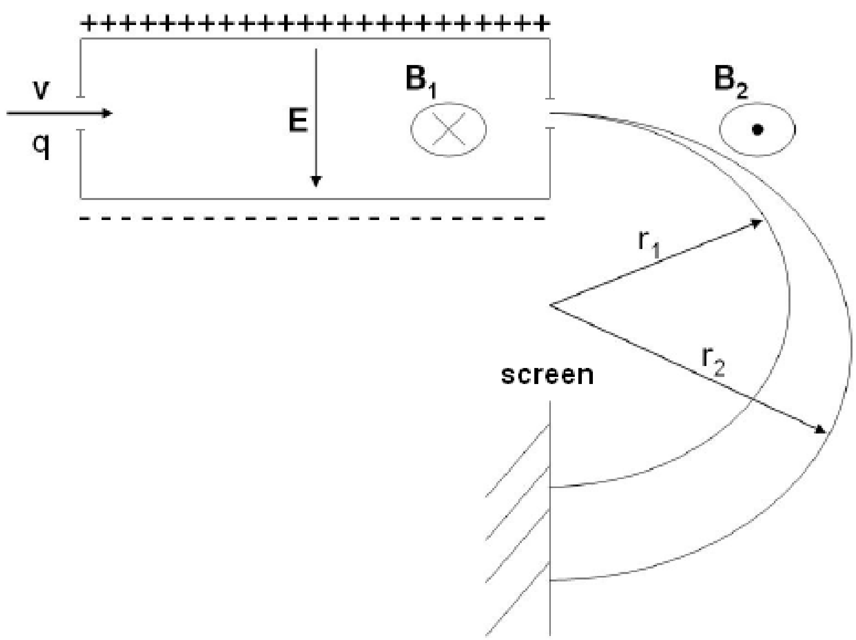

Figure 1: Diagram of a Dempster-type mass spectrometer

Solution:

The first unit is the velocity selector, which only lets ions with a certain velocity to pass through. According to Newton's laws the condition for straight crossing is that the resultant force should be zero:

$$\underline{F}_e + \underline{F}_m = q\underline{E} + q\underline{v} \times \underline{B}_1 = 0$$

Since $\underline{E}$, $\underline{v}$ and $\underline{B}_1$ are perpendicular to each other:

$$qE - qvB_1 = 0$$

therefore:

$$v = \frac{E}{B_1}$$

which means that the velocity selector works independently from the charges of the ions.

The centripetal force which maintains the circular path arises from the second magnetic field:

$$\underline{F}_{cp} = q\underline{v} \times \underline{B}_2$$

we apply the definition of $\underline{F}_{\mathrm{cp}}$ and use that $\underline{v}$ is also perpendicular to $\underline{B}_2$:

$$\frac{mv^2}{r} = qvB_2$$

therefore the radius of the circle will be:

$$r = \frac{mv}{qB_2}$$

The distance between the impact points after passing one semicircle will be the following:

$$2(r_2 - r_1) = 2\left(\frac{m_1 v}{qB_2} - \frac{m_2 v}{qB_2}\right) = \frac{2v}{qB_2}(m_1 - m_2) = \frac{2\Delta mv}{qB_2}$$

substituting the velocity in the equation we get:

$$2(r_2 - r_1) = \frac{2\Delta m}{qB_2} \cdot \frac{E}{B_1} = 0.02498 \text{ m} = 2.498 \text{ cm}$$

Exercise 2: Mass-doublet method

In knowledge of m(^{12}C) and m(^{1}H) determine the mass of the ^{14}N atom. Suppose that the nitrogen is in gaseous form N_2.

1) Find a suitable hydrocarbon compound for the comparison.

2) Calculate the ^{14}N mass given that we measured Δm=0.02515220 ± 0.00000003 u.

Solution:

1) The molar mass of the N_2 gas is 28 g/mol, so we need to find a hydrocarbon compound with a similar molar mass. The only suitable hydrocarbon compound is C_2H_4 (ethene).

2) The measured mass difference can be expressed in the following way:

$$\Delta m = m(C_2H_4) - m(N_2) = 2m(^{12}C) + 4m(^{1}H) - 2m(^{14}N)$$

after rearranging we get the mass of the ^{14}N atom:

$$m(^{14}N) = \frac{1}{2}\left[2m(^{12}C) + 4m(^{1}H) - \Delta m\right] = 14.00307396 \pm 0.00000002 \text{ u}$$

Exercise 3: A brief introduction to ion optics

As we have already seen in the lecture, a mass spectrograph is an instrument that separates and simultaneously focuses ions, along a focal plane, of different mass/charge ratios that are diverging in direction and that have a variable velocity. In this example we aim to give a brief introduction to the methodology with which mass spectrographs can be described and optimized. Further information can be found under the following link:

https://www.uni-ulm.de/fileadmin/website_uni_ulm/nawi.inst.220/lehre/Atomphysik_SS2009/massenspektrometer_Matrizenrechnung.pdf

The different segments of the mass spectrograph are represented with so called transfer matrices, similarly to those used in geometric optics. The ion beam can be described with four parameters, namely the x image width, $a \approx \tan\alpha$ angle of divergence, ∂E energy dispersion and ∂M mass dispersion (see Figure 2).

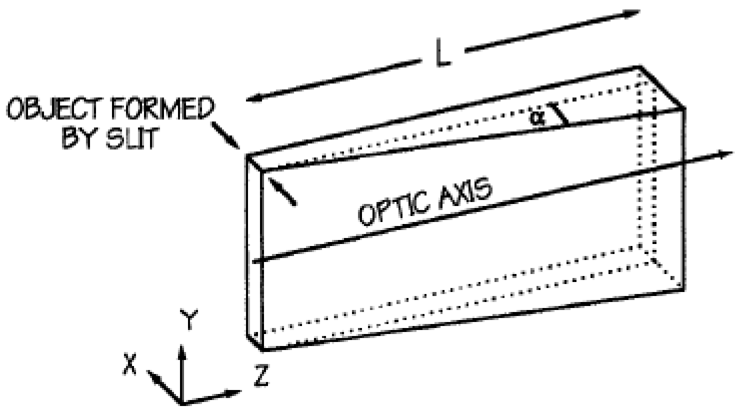

Figure 2: Diagram of an ion beam in field-free region of width x and angular divergence α

A general, first-order matrix is shown below:

$$\begin{bmatrix} x \\ a \\ \partial E \\ \partial M \end{bmatrix} = \begin{bmatrix} x\,|\,x_0 & x\,|\,a_0 & x\,|\,\partial E_0 & x\,|\,\partial M_0 \\ a\,|\,x_0 & a\,|\,a_0 & a\,|\,\partial E_0 & a\,|\,\partial M_0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ a_0 \\ \partial E_0 \\ \partial M_0 \end{bmatrix}$$

In the following, we will present three segment types in mass spectrometers: the drift length, the electric sector and the magnetic sector.

1) Drift length:

A drift length is a field-free region of length L, where no acceleration, deceleration, or focusing of the ion beam occurs. The transfer matrix for the drift length is the following:

$$[DL] = \begin{bmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Electric sector (ESA):

An electric sector is composed of a section of two circular plates with a center line of curvature equal to radius R_e and angle Φ_e (see Figure 3). The potentials on the plates are equal but opposite, although small voltage differences can be used to alter the beam direction. The transfer matrix for an electric sector or ESA is the following:

$$[ESA] = \begin{bmatrix} \cos(\sqrt{2}\Phi_e) & \frac{R_e}{\sqrt{2}}\sin(\sqrt{2}\Phi_e) & \frac{R_e}{2}(1-\cos(\sqrt{2}\Phi_e)) & 0 \\ -\frac{\sqrt{2}}{R_e}\sin(\sqrt{2}\Phi_e) & \cos(\sqrt{2}\Phi_e) & \frac{1}{\sqrt{2}}\sin(\sqrt{2}\Phi_e) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

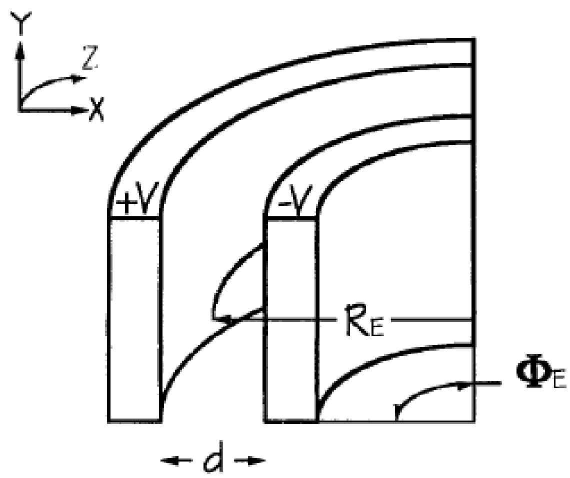

Figure 3: Diagram of an electric sector (ESA)

3) Magnetic sector (MAG):

A magnetic sector consists of a north and a south magnetic pole separated by a narrow distance, and it is described by the Φ_m angle and the R_m radius of deflection (see Figure 4). The transfer matrix for the magnetic sector or MAG is the following (note that the ion beam should enter and exit the magnetic sector in a direction normal to the pole faces):

$$[MAG] = \begin{bmatrix} \cos(\Phi_m) & R_m \sin(\Phi_m) & \frac{R_m}{2}(1-\cos(\Phi_m)) & \frac{R_m}{2}(1-\cos(\Phi_m)) \\ -\frac{1}{R_m}\sin(\Phi_m) & \cos(\Phi_m) & \frac{1}{2}\sin(\Phi_m) & \frac{1}{2}\sin(\Phi_m) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

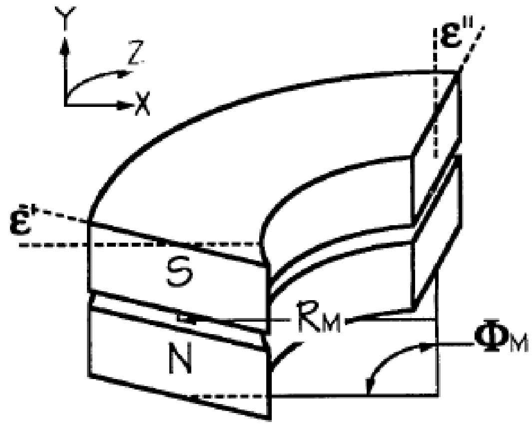

Figure 4: Diagram of a magnetic sector (MAG)

In knowledge of these three instruments, we are able to describe numerous mass spectrograph geometries, of which the Bainbridge-Jordan mass spectrograph is illustrated in Figure 5.

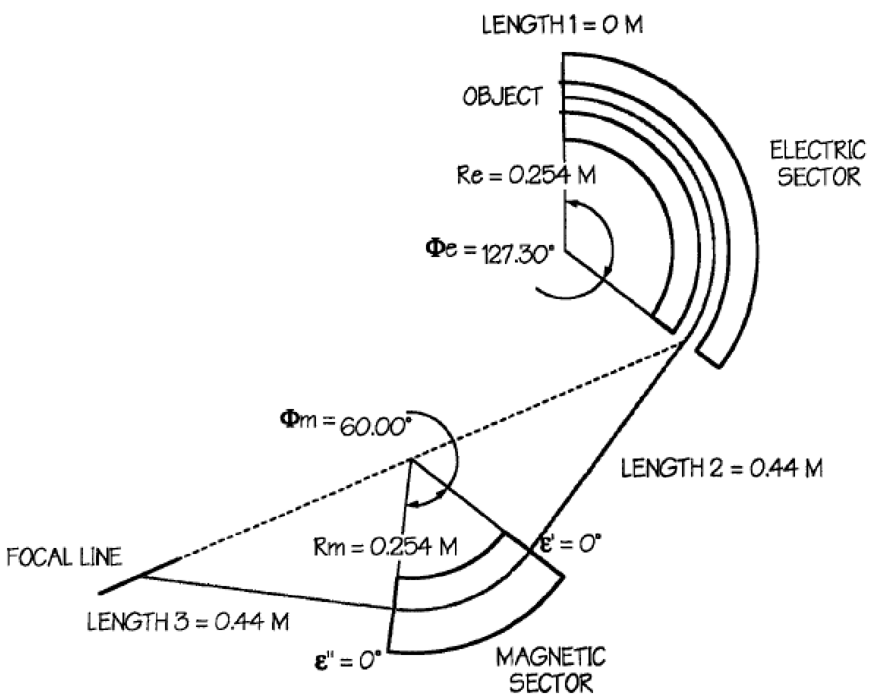

Figure 5: Diagram of the Bainbridge-Jordan mass spectrograph geometry

The Bainbridge-Jordan mass spectrograph geometry consists of a $\pi/\sqrt{2}$ ESA without an initial drift length. Notice that from the [ESA] transfer matrix, it follows that an ion beam with a slight divergence will cross itself at every $\pi/\sqrt{2}$ angle in an electric sector (unlike in a magnetic sector, where an ion beam will cross at every π angle). The magnetic sector focuses the beam using Barber's Rule, i.e. the object, center of deflection an image lie along a straight line.