Nuclear Physics
(13th lecture)

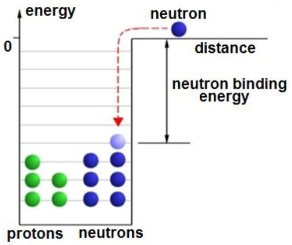
- **Cross sections of special neutron-induced reactions**

NUCLEAR FISSION

- **Mechanism and characteristics of nuclear fission.**
  - **The fission process**
  - **Mass distribution of the fragments**
  - **Energy balance**
  - **Fission barrier**
  - **Fission neutrons: prompt and delayed neutrons**
- **Nuclear chain reaction**
  - **Time behaviour of the nuclear chain reaction, criticality**
  - **Methods for achieving self-sustaining chain reaction**

Cross sections of special neutron-induced reactions

Capture of low-energy neutrons



Usually it is an exothermic process, because of the binding energy of the neutron.

The neutron is neutral, therefore there is no activation energy ⟶ also neutrons with very small kinetic energy can induce this reaction.

The reaction probability is proportional to the time what the neutron and the nucleus spend close together

This is the *1/v* cross section $\sigma \sim t = \frac{2R}{v}$

2R ← radius of the nucleus

v ← velocity of the neutron

How does it look in a $\log(\sigma) - \log(E_n)$ graph?

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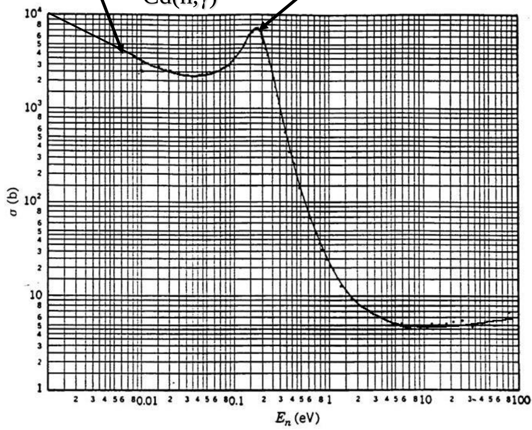
1. example: neutron capture cross section of Cd $\sigma_{n,\gamma}$:

1/v region

resonance

Cd(n,γ)



Cadmium is an important n-absorber and shielding material!!

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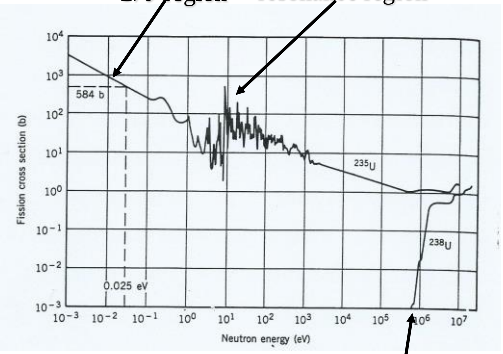
2. Example Fission cross-section for uranium isotopes

1/v region

resonance region



energy threshold for ^{238}U(n,f)

Nuclear energy

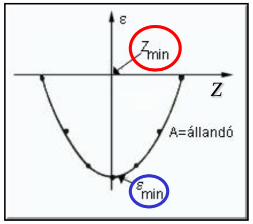
Reminder: Binding energy of nuclei (Weizsäcker)

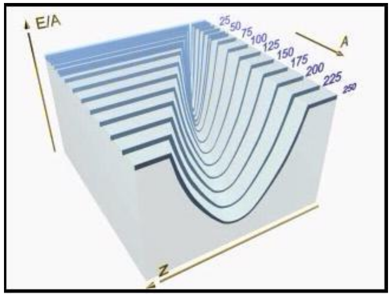
$$\boxed{B = b_V A - b_F \cdot A^{2/3} - b_C \cdot \frac{Z^2}{A^{1/3}} - b_A \cdot \frac{(N-Z)^2}{A} + b_P \cdot \delta \cdot A^{-3/4}}$$

$$\varepsilon = \frac{E}{A} = -\frac{B}{A}$$

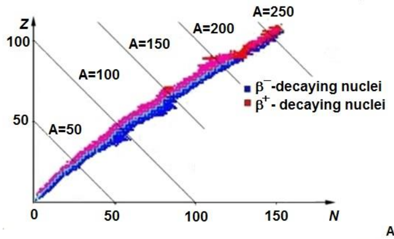
(average energy of a nucleon)
(energy = – binding energy)

The A = const. cuts are parables!





The „energy valley": $\varepsilon(N,Z)$



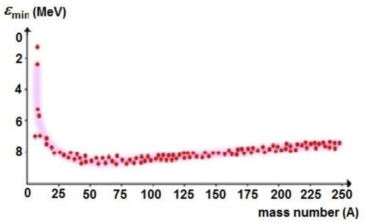
The place of Z_{min} on the (N,Z) „map" is between the blue and purple region

This way the radioactive decays could be explained!

ε_{min} in function of mass number (A)

This helps to understand the nuclear energy production!

Average energy of a nucleon along the stability line



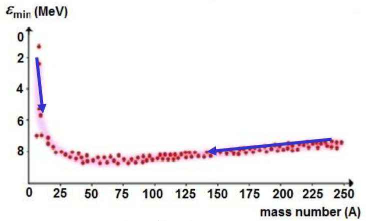
Two ways of producing nuclear energy

Fusion
(make larger nuclei from small ones)

Fission
(make smaller nuclei from large ones)

For example
$^2H + ^3H \longrightarrow ^4He + n$
(Q = 17,6 MeV)

Average energy of a nucleon along the stability line



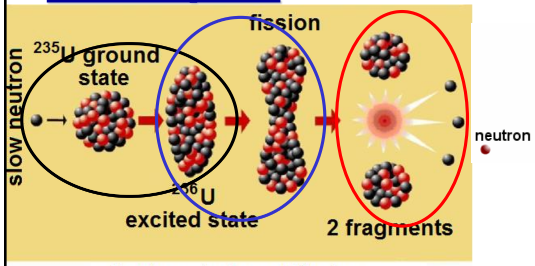
For the fusion:
- large gain in energy/nucleon (~ 2 … 5 MeV/nucleon)
- few nucleons (~ 2 ... 5),

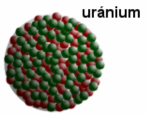
Total ~ 2 – 18 MeV

For the fission:
- small gain in energy/nucleon (~0,85 MeV/nucleon)
- many nucleons (~235),

Total ~ 200 MeV

The fission process





Neutron induced fission

1) The nucleus gets in an excited state
(for example by capturing a neutron)

2) The shape gets deformed, a thinner neck forms

3) Scissions into two parts, a few neutrons get emitted

Heavy transuranium elements may fission spontaneously, no need to excite them. Then the first step may be missing.

The fission reaction:

$^{235}U + n \longrightarrow \text{Fragment}_1 + \text{Fragment}_2 + \boxed{2{,}4\ n}$

The composition (Z,A) of the fragments cannot be given more precisely, since the fission process is stochastic also in this respect !! (This is why the average number of emitted neutrons is not an integer)

Mass distribution of the fragments:

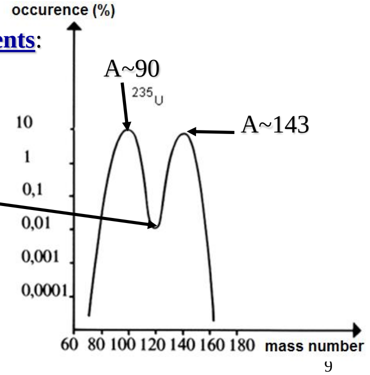
If the fission was symmetric:

$$^{236}_{92}U \rightarrow {}^{118}_{46}Pd + {}^{118}_{46}Pd$$

The production of A~120 fragments ~1000 less probable

The fission is most likely asymmetric ! ($^{235}U + n_{therm}$)

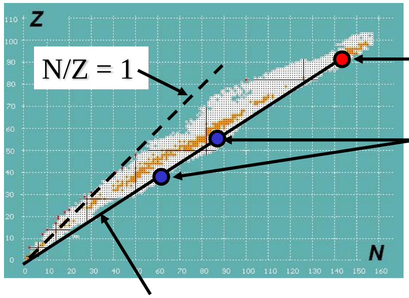
This distribution depends on the fissile material and also on the energy of the neutrons!



Fission on the (N,Z) „map”

$^{236}_{92}U_{144}$

N/Z = 144/92 = 1,56



N/Z = 1,56

Starting point (N/Z ~ 1,56) („stable” nucleus T~4,5·10⁹ y)

End points if the ratio remains N/Z ~ 1,56 (asymmetric fission)

NEUTRONRICH nuclei

Highly radioactive(β⁻ decays) fission products

Neutron emission necessary ($\bar{\nu}$ ~ 2,4)

Energy balance of the fission $^{235}U(n,f)$

The total energy will be released through several different processes. This influences the place and the time of heat production

🟥🟪	The kinetic energy of the fragments	168 MeV (82,0 %)
🟥	The energy of the β-particles from the fragments	8 MeV (3,9 %)
🟦🟪	Energy of the neutrons emitted in the fission	5 MeV (2,4 %)
🟦🟪	Energy of the prompt γ-photons	7 MeV (3,4 %)
🟦	Energy of the γ –radiation of the fragments	7 MeV (3,4 %)
🟩	Energy of the antineutrinos emitted by the β-decays of the fragments	10 MeV (4,9 %)
	TOTAL	205 MeV (100%)

🟥 Short range (in the fuel, or close to it)
🟦 Medium range (coolant, reactor vessel, biological shield)
🟩 Very long range (leaves the reactor)
🟪 Prompt (in time of the fission)

Example: suppose that the ^{236}U nucleus fissions the following way:

$^{236}U \longrightarrow {}^{90}Kr + {}^{143}Ba + 3n$

Determine, how „far” are the fragments at the scission point, when only Coulomb-forces act.

Assume, that their total kinetic energy is 168 MeV.

Solution:

At the „scission point” they have only Coulomb potential energy, this will turn into their kinetic energy:

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Z_1Z_2e^2}{d} = 168\cdot 1{,}6\cdot 10^{-13}\ \text{J}$$

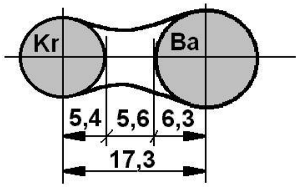
Here Z_1=36 (Kr), Z_2=56 (Ba)

From this we get: d ~ 17,3 fm

The radius of both nuclei using $R = r_0\sqrt[3]{A}$

R_{Kr} = 5,4 fm, R_{Ba} = 6,3 fm

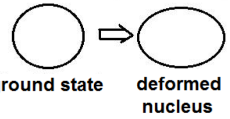
The geometry of the „scission”:



Fission barrier

The fission process begins with a small deformation.

(deformation parameter: ε)



How much energy is needed for that?

The needed energy will be determined using the Weizsäcker-formula:

$$B = \boxed{b_V A} - b_F \cdot A^{2/3} - b_C \cdot \frac{Z^2}{A^{1/3}} - \boxed{b_A \cdot \frac{(N-Z)^2}{A} + b_P \cdot \delta \cdot A^{-3/4}}$$

During the deformation

- volume = constant,
- asymmetry, even-odd = constant
- surface → increasing $b_F A^{2/3} \Rightarrow b_F A^{2/3}(1 + a_F \cdot \varepsilon)$
- Coulomb-energy → decreasing $b_C \frac{Z^2}{A^{1/3}} \Rightarrow b_C \frac{Z^2}{A^{1/3}}(1 - a_C \cdot \varepsilon)$
  (protons get farther apart)

The factors can be calculated, the result:

$a_F \sim 0{,}025$ and $a_C \sim 0{,}012$.

The change in the energy:

$$\Delta E = \left(0{,}025 \cdot b_F \cdot A^{2/3} - 0{,}012 \cdot b_C \frac{Z^2}{A^{1/3}}\right) \cdot \varepsilon$$

If $\Delta E < 0$, then the nucleus fissions spontaneously for any small ε deformation!

$$\left(0{,}025 \cdot b_F \cdot A^{2/3} - 0{,}012 \cdot b_C \frac{Z^2}{A^{1/3}}\right) < 0 \text{ follows: } \boxed{\frac{Z^2}{A} \geq \frac{0{,}025 \cdot b_F}{0{,}012 \cdot b_C} \approx 54}$$

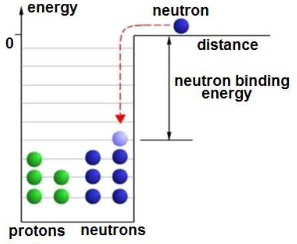
(since b_F=2,85 ·10^{-12} J, b_C=0,11 ·10^{-12} J)

For large nuclei $Z/A \sim 0{,}39$, therefore at $Z > 136$ there is no „activation energy” for fission. (In fact, already around Z~110, the activation energy is so small, that the nuclei undergo fission spontaneously quite quickly by tunnel effect.)

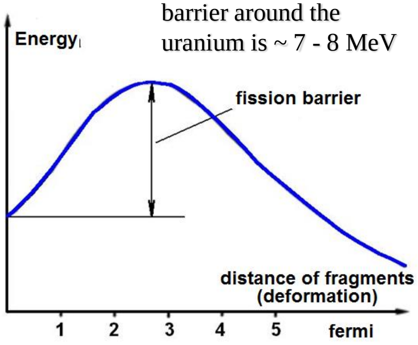
The Periodical System ends because of the fission!!!

For $Z < 136$ small deformations need some energy ($\Delta E > 0$) this way a fission barrier is formed

How can a slow (low energy) neutron make the ^{235}U nucleus fission?



The height of the fission barrier around the uranium is ~ 7 - 8 MeV



The binding energy of the neutron covers the activation energy of the fission!

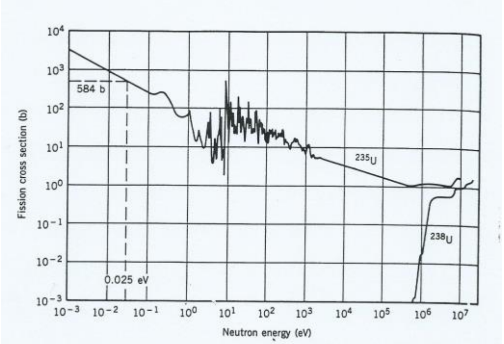
Importance of the pairing energy:

From ^{235}U + n even-even nucleus (^{236}U) ⟶ larger Q

From ^{238}U + n even-odd nucleus (^{239}U) ⟶ smaller Q

therefore ^{235}U + n fissions by low energy neutron,

for ^{238}U +n there is an energy threshold (~0,8 MeV)



The fission neutrons

We saw: the fragments have too many neutrons after the fission

⟶ neutron emission is inevitable

However, the number of the emitted neutrons is not constant, it fluctuates around a mean value. It can be well approximated by a Gaussian

The mean value depends on the fissile nucleus, and on the energy of the fission-inducing neutron.

FWHM: ~ 2,5 (does not depend on the nucleus)

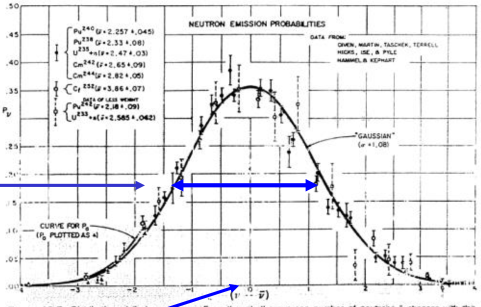


Figure 13.7 Distribution of fission neutrons. Even though the average number of neutrons $\bar{\nu}$ changes with the fissioning nucleus, the distribution about the average is independent of the original nucleus. From J. Terrell, in *Physics and Chemistry of Fission*, Vol. 2 (Vienna: IAEA, 1965), p. 3.

Mean value: $\nu = 2{,}43$ ($^{235}U+n_{th}$ fission)

Timely appearance of neutrons

(relative to the time of fission)

a) Prompt neutrons

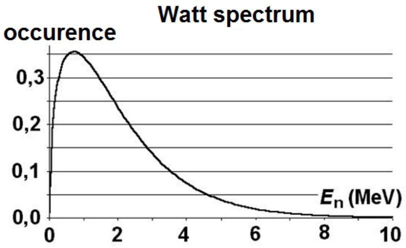
The largest part of the neutrons is emitted promptly just at the time of the fission. These are the prompt neutrons ($t < 10^{-16}$ s).

Their mean energy is ~ 2 MeV

Energy-spectrum:
„Watt-spectrum"

$$N(E_n) \sim 0{,}484 \cdot e^{-E_n/E_0} \cdot \sinh\sqrt{2\frac{E_n}{E_0}}$$

(E_0 = 1 MeV)



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b) „Delayed" neutrons

We saw: the fragments have too many neutrons after the fission

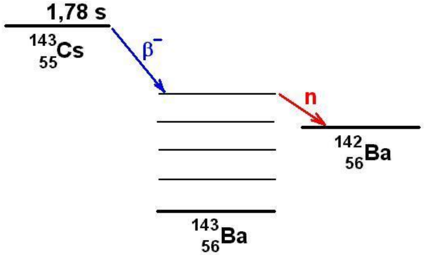
These all are β^--decaying.

In some cases after the β^-- decay the daughter nucleus is in such highly excited state, that neutron-emission is energetically possible

⟶ delayed neutron

A possible example:

(the neutron is emitted with1,78 s half life!)



There are many such kind of decays with different half lives, different yields

The naming convention:

The delayed neutrons are grouped into 6 groups (according to half lives)

	E_n(MeV)	T_i(s)	β_i(%)	Typical precursors
1	0,25	56	0,020	^{87}Br, ^{142}Cs
2	0,56	23	0,143	^{88}Br, ^{137}I
3	0,43	6,2	0,128	^{89}Br, ^{138}I
4	0,62	2,3	0,255	^{94}Kr, ^{139}I, ^{143}Cs
5	0,42	0,6	0,074	^{140}I, ^{145}Cs
6	0,51	0,2	0,030	^{87}As, ^{143}Xe

Total: $\beta = 0{,}65$ %

The delayed neutron ratio: $\beta = \frac{\text{(delayed n)}}{\text{(total n)}} \sim \frac{\text{(delayed n)}}{\text{(prompt n)}}$

The appearance of the delayed neutrons after the fission:

$$N(t) = \sum_{i=1}^{6} \beta_i \cdot e^{-\ln 2 \cdot \frac{t}{T_i}}$$

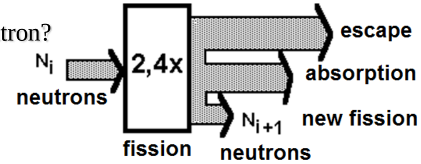
Their role is very important in the control of the chain reaction!!

Chain reaction with neutrons

„**Neutron-balance**”

What can happen with a neutron?

- Escapes from the reactor
- Gets absorbed
- Induces new fission



Neutron „generations”

$N_1, N_2, N_3, \ldots N_i, N_{i+1}, \ldots$

N_i is the number of neutrons inducing fission in the i^{th} generation

Neutron multiplication factor: $\boxed{k_{eff} = \dfrac{N_{i+1}}{N_i}}$ (definition)

$$\text{If} \begin{cases} k_{eff} < 1, & \text{the chain reaction is decreasing („sub critical”)} \\ k_{eff} = 1, & \text{the chain reaction is stationary („critical”)} \\ k_{eff} > 1, & \text{the chain reaction is increasing („supercritical”)} \end{cases}$$

Time behaviour of the chain reaction

Starting point: $\boxed{k_{eff} = \dfrac{N_{i+1}}{N_i}}$

Some algebra: $k_{eff} - 1 = \dfrac{N_{i+1}}{N_i} - 1 = \dfrac{N_{i+1} - N_i}{N_i} = \dfrac{\Delta N}{N_i}$

The (average) time between two generations: $\Delta t = \ell$ (generation time)

Multiplying the two equations: $\Delta t \cdot (k_{eff} - 1) = \ell \cdot \dfrac{\Delta N}{N}$

We get: $\dfrac{\Delta N}{\Delta t} = \left(\dfrac{k_{eff} - 1}{\ell}\right) \cdot N$

Using $\Delta t \to 0$ we get $\dfrac{dN}{dt} = \left(\dfrac{k_{eff} - 1}{\ell}\right) \cdot N(t)$

The solution of this differential equation is: $\boxed{N(t) = N_0 \cdot e^{\left(\frac{k_{eff}-1}{\ell}\right)t}}$

$\boxed{N(t) = N_0 \cdot e^{\left(\frac{k_{eff}-1}{\ell}\right)t}}$

Obviously for $k_{eff} = 1$ we get $N(t) = N_0 =$ constant,

For $k_{eff} > 1$ the $N(t)$ increases exponential in time,

For $k_{eff} < 1$ the $N(t)$ decreases exponential in time.

The changing rate is determined by $\left(\dfrac{k_{eff} - 1}{\ell}\right)$

For prompt neutrons the generation time is $\ell \sim 10^{-4}$ s

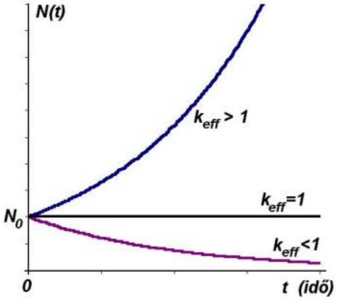
Example:

Suppose that $k_{eff} = 1{,}001$

The number of neutrons (and also the power of the reactor) changes in one second by…

$N(1) = N_0 \cdot e^{\frac{1{,}001-1}{0{,}0001} \cdot 1} =$

$= N_0 \cdot e^{10} = 22026 \cdot N_0$



Uncontrollable! (prompt-critical)

The generation time of the delayed neutrons will be extended by the half-life of the precursor → it can reach even several seconds!

The role of the delayed neutrons: increase the effective generation time!

The system can be controlled, if $k_{eff} < 1$ without the delayed neutrons!

Therefore, $k_{eff} < 1 + \beta = 1{,}0065$ should always be fulfilled!

Reactivity: $\boxed{\rho = \dfrac{k_{eff} - 1}{k_{eff}}}$ (definition)

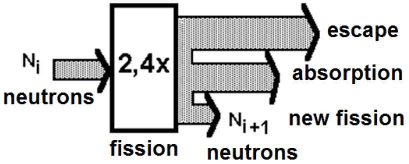
For a prompt-critical system: $k_{eff} = 1 + \beta$, therefore its reactivity:

$\rho = \dfrac{1 + \beta - 1}{1 + \beta} \approx \beta$ (since $1 + \beta = 1{,}0065 \sim 1$)

Commonly used unit of the reactivity is the \$ (dollar), which is the reactivity in delayed neutron units.

The reactivity is 1\$, if $\dfrac{\rho}{\beta} = 1$

Methods for achieving self-sustaining chain reaction



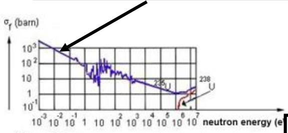
escape (occurs on the surface)

absorption (^{238}U only absorbs, no fission)

$$k_{eff} = \frac{N_{i+1}}{N_i}$$

The fraction of „new fission” should be increased.
There are several methods for that

Slow down the neutrons
(fission cross section increases)



^{235}U and ^{238}U fission cross section depends on the neutron energy

Decrease the fraction
of „absorption”

increase the ratio
^{235}U/^{238}U (**enrichment)**

Decrease the fraction
of the „escape”

Large **size**
(surface/volume)
decreases
Critical mass