Nuclear Physics
(12th lecture)

NUCLEAR REACTIONS

- **Nuclear reactions. Conserved quantities. Reaction energy**
- **Kinematics, laboratory and centre of mass (CM) systems**
- **Microscopic and macroscopic cross sections**
- **Two additivities of the cross sections.**
- **Differential cross-sections.**
- **Excitation functions.**
- **Nuclear reaction mechanisms**
- **Direct reactions: knock-out, pick-up, stripping**
- **Compound reactions and resonances**

Nuclear reactions

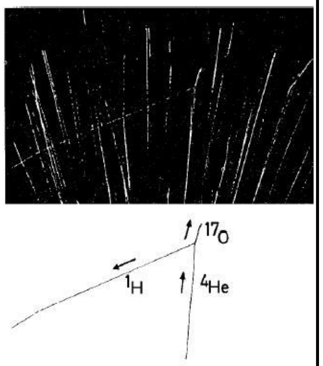
Importance: Most of our knowledge about nuclei is gained from studies on nuclear reactions

The first artificially created (observed) nuclear reaction is from
E. Rutherford (1919)
Observation: cloud chamber

$${}^{14}_{7}N + {}^{4}_{2}He \to {}^{17}_{8}O + {}^{1}_{1}H$$

α-particle from radium

Nitrogen, gas in the cloud chamber



Picture: Blackett and Lees

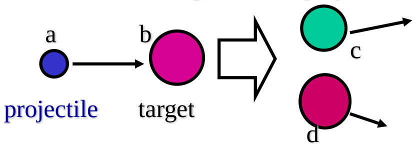
Generally:

a + b ⟶ c + d + e + …

Initial state ⟶ Final state

(reaction of more than two particles is not likely)

Often one of the interacting particles is at rest in the laboratory ⟶ target,
The other is moving ⟶ projectile



Initial state

Notation: b (a, c) d

target (at rest)

Final state

Scatterings: special nuclear reactions

where a = c, (and b = d), which means that the type (composition) of the particles does not change.

Elastic scattering: the particles do not get excited,
the total kinetic energy is conserved

Inelastic scattering: at least one of the particles get excited
(then γ–decay), the total kinetic energy is NOT conserved.

Examples	Name	Notations
$n + {}^{235}_{92}U \to {}^{235}_{92}U + n'$	elastic neutron scattering (n,n’)	${}^{235}_{92}U(n,n'){}^{235}_{92}U$
$n + {}^{235}_{92}U \to {}^{235}_{92}U + n' + \gamma$	inelastic n-scattering (n,n’γ)	${}^{235}_{92}U(n,n'\gamma){}^{235}_{92}U$
$n + {}^{235}_{92}U \to {}^{236}_{92}U + \gamma$	n-capture with γ-emission, radiating capture, (n,γ) reaction	${}^{235}_{92}U(n,\gamma){}^{236}_{92}U$
$\alpha + {}^{9}_{4}Be \to {}^{12}_{6}C + n$	α-induced n-emission, (α,n) reaction	${}^{9}_{4}Be(\alpha,n){}^{12}_{6}C$
$n + {}^{59}_{27}Co \to {}^{58}_{27}Co + 2n$	(n,2n) reaction	${}^{59}_{27}Co(n,2n){}^{58}_{27}Co$

Conserved quantities:

- nucleon number (A) (baryon charge) → $^{14}_{7}\mathrm{N}+^{4}_{2}\mathrm{He}\rightarrow^{17}_{8}\mathrm{O}+^{1}_{1}\mathrm{H}$
- electric charge →
- leptonic charge (if electron, positron, neutrino etc. participates in the reaction)

$\left.\begin{array}{l}\text{• energy (taking into account } E = mc^2\text{)}\\ \text{• momentum}\\ \text{• angular momentum}\end{array}\right\}$ Kinematical parameters

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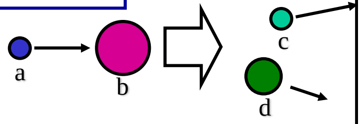
Energy balance

The nuclear reaction: $\boxed{a + b \longrightarrow c + d}$

M_a, M_b etc. are the masses,
T_a, T_b etc. are the kinetic energies



The energy conservation:

$$(M_a\cdot c^2+T_a)+(M_b\cdot c^2+T_b) = (M_c\cdot c^2+T_c)+(M_d\cdot c^2+T_d)$$

Arranging the masses to the left hand side:

$$\boxed{(M_a+M_b-M_c-M_d)\cdot c^2 = T_c+T_d-T_a-T_b= Q} \qquad (*)$$

Q is called: reaction energy

Its physical meaning is clear from the second equation:

$$(T_c + T_d)-(T_a + T_b)= Q$$

$Q > 0$ ⟶ The reaction produces kinetic energy
(exothermic, exoergic, „energy producing” reaction)

$Q < 0$ ⟶ The reaction consumes kinetic energy
(endotherm, endoergic, „energy consuming” reaction)

$Q = 0$ ⟶ The reaction conserves the kinetic energy
(For example elastic scattering)

$$\boxed{(M_a+M_b-M_c-M_d)\cdot c^2 = T_c+T_d-(T_a+T_b)= Q} \qquad (*)$$

Energy threshold for endotherm reactions $(Q<0)$.
Since $T_c + T_d \geq 0$, therefore $(T_a + T_b) \geq -Q > 0$.
The initial kinetic energy of the particles should be at least at this level, for the reaction to occur!

The reaction energy and the masses of the particles:

From the (*) equation $Q = (M_a+M_b-M_c-M_d)\cdot c^2$

This way the reaction energy can be calculated!

Here M_a, M_b etc. are not necessary the ground state rest masses of the particles! For example if particle d was formed in an excited state with E_x energy, then $M_d = \underset{\substack{\uparrow\\ \text{ground state rest mass}}}{M_d(0)} + E_x/c^2$

Activation energy (for example at electrically charged particles)

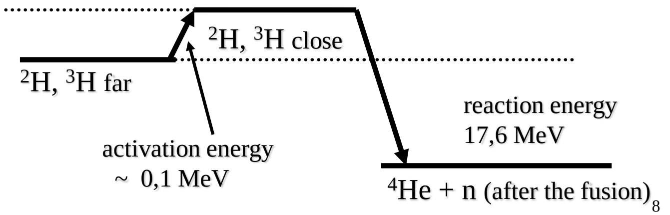
One of the main reactions of the fusion energy production:

$$^{2}_{1}\mathrm{H}+^{3}_{1}\mathrm{H}\rightarrow^{4}_{2}\mathrm{He}+\mathrm{n}+17{,}6\ \mathrm{MeV}$$

This reaction does not occur spontaneously, although it is exothermic!
Cause: the nuclear interaction has short range, ⟶ the reaction partners have to get close together.

They need some (kinetic) energy, because of the Coulomb-repulsion!

The energy conditions (without the kinetic energies):



Fundamentals of the kinematic description of nuclear reactions

Starting point: energy- and momentum-conservation.

Energy-conservation has been treated already.

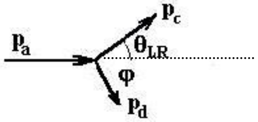
Momentum-conservation: $\boxed{p_a + p_b = p_c + p_d}$ vector-equation!!

Choice of coordinate system:

Laboratory system ⟺ **Centre of mass system (CM system)**

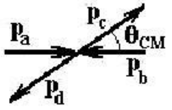
(the data of the measurements are produced here)

The target nucleus is usually at rest, i.e. $p_b = 0$, or $p_a = p_c + p_d$



(This is the „natural" coordinate system)

$p_a + p_b = p_c + p_d = 0$



Naturally $\theta_{LR} \neq \theta_{CM}$

Additional note to the energy threshold

We saw: $(T_a + T_b) \geq -Q$ .However, this is valid only in CM-system, since here the total momentum and kinetic energy of the system is 0.

These T_a and T_b kinetic energies are the energies in CM-system.

In laboratory system the total momentum of the system is not zero, and this has to be conserved after the reaction!

Suppose, that the b target nucleus is at rest: $T_b = 0$.

The energy threshold for an endotherm reaction: $\boxed{T_a > -Q\left(1 + \frac{M_a}{M_b}\right)}$

(try to demonstrate it at home!)

Here T_a is the kinetic energy in the laboratory system

If $M_a >> M_b$, then $T_a >> -Q$!

(Inverse kinematics: the projectile is heavier than the target)

For colliding beams: laboratory system = CM system

The probability of nuclear reactions

The nuclear reactions are stochastic processes.

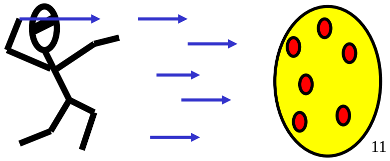
(Remember: the radioactive decay was also a stochastic process!)

They can be described by statistical laws.

Model: Consider a „dart" board of F = 1 m^2 surface, where N = 100 pieces of target area are scattered randomly. The surface of each target area is σ = 1 cm^2. A blind-folded player throws darts on the board. During 1 hour altogether 200 darts hit the board (n = 200/h). How many target hits can be expected in an hour?

$$R = \frac{200}{10^4\ \text{cm}^2} \cdot 100 \cdot 1\,\text{cm}^2 = 2$$

$$R = \frac{n}{\text{F}} \cdot N \cdot \sigma$$



Denote the n/F by Φ.

$$R = \frac{n}{\text{F}} \cdot N \cdot \sigma$$

Its meaning: the number of particles flying through unit area in unit time.

The name of Φ: scalar flux

Its unit is: 1/(time·area) = [1/(s·cm^2)]

With that, the number of „hits" during unit time: $\boxed{R = \phi \cdot N \cdot \sigma}$

The name of R is: reaction rate

Its unit is: 1/time = [1/s]

The name of σ is: total, microscopic cross section

Its unit is: area = [cm^2]

Order of magnitude of the surface of nuclei:

$$\left(10^{-14}\ \text{m}\right)^2 = 10^{-28}\ \text{m}^2 = 10^{-24}\ \text{cm}^2 \qquad \boxed{1\ \text{b} = 10^{-24}\ \text{cm}^2}$$

barn

The general definition of the cross section: $\boxed{\sigma = \frac{R}{N \cdot \phi}}$

Very important:

Although it was introduced using an illustrative model, this is NOT the actual geometrical surface of the nucleus!!!
Even, this is NOT the common geometrical area of target + projectile system!

Every nuclear reaction has different cross section!

(since R determines, which reaction is considered)

Example: the cross sections of the following two reactions are different, although the target and the projectile are the same!

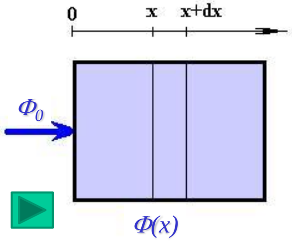
$n + {}^{235}U \longrightarrow {}^{236}U + \gamma$ (radiative capture)

$n + {}^{235}U \longrightarrow {}^{235}U + n' + \gamma$ (inelastic scattering)

Moreover, the cross section usually depends also on the energy of the projectile σ = σ(E) (see later)

The microscopic cross section (σ) is a measure of the probability of a nuclear reaction. Its unit is barn [b] (=10^{-24} cm^2)

Measuring the total microscopic cross section by transmission



If Φ_0 flux falls in parallel beam onto a target slab (of surface F), this will decrease somewhat in x distance, since the particles get absorbed or scattered out. There remains only $\Phi(x)$ flux at x depth.

If ρ is the density of the target nuclei (number/cm^3), then the number of target nuclei in a layer of dx thickness is $N=\rho \cdot F \cdot dx$.
The number of any reactions in this layer (during unit time) is:

$$R = \phi(x) \cdot N \cdot \sigma_t = \phi(x) \cdot (\rho \cdot F \cdot dx) \cdot \sigma_t$$

The decrease in flux in this layer (R/F): $d\phi = -(\rho \cdot \sigma_t) \cdot \phi(x) \cdot dx$

From this we get: $\frac{d\phi}{dx} = -(\rho \cdot \sigma_t) \cdot \phi(x)$

The solution: $\boxed{\phi(x) = \phi_0 e^{-(\rho \cdot \sigma_t) \cdot x}}$

This way the total microscopic cross section can be measured

Macroscopic cross section

Definition $\boxed{\Sigma = \rho \cdot \sigma}$ $\left(\frac{1}{\text{cm}^3} \cdot \text{cm}^2\right)$

The name of Σ is: macroscopic cross section.
The unit of the macroscopic cross section is: 1/distance (!!)

Therefore the above example can be written as: $\boxed{\phi(x) = \phi_0 e^{-\Sigma_t \cdot x}}$

The total macroscopic cross section can be measured by measuring the decrease in flux.

The two additivities of the cross sections

Remember: every nuclear reaction has its own cross section: $\boxed{\sigma = \frac{R}{N \cdot \phi}}$

I. Additivity: same reaction partners, different reactions

If the partners are the same, then N and Φ are also the same for the different reactions, only the reaction rates are different (R_i, i=1,2,3..).

Then $R_{total} = R_1 + R_2 + \ldots$

Therefore the total microscopic cross section (σ_t):

$$\sigma_t = \frac{R_{total}}{N \cdot \phi} = \frac{R_1 + R_2 + \ldots}{N \cdot \phi} = \frac{R_1}{N \cdot \phi} + \frac{R_2}{N \cdot \phi} = \sigma_1 + \sigma_2 + \ldots$$

Summarized: $\boxed{\sigma_t = \sigma_1 + \sigma_2 + \ldots}$

Multiplying both sides with the target nucleus density: $\boxed{\Sigma_t = \Sigma_1 + \Sigma_2 + \ldots}$

Obvious condition: all, mutually exclusive reactions should be listed in the right hand side

II. Additivity: composite target (composed of different elements)

Denote the nuclear densities (number of different nuclei per unit volume) by $\rho_1, \rho_2, \rho_3, \ldots \rho_N$

The different materials act on the projectile beam independently, i.e.

$$\boxed{\Sigma_t(\text{result}) = \Sigma_t(1) + \Sigma_t(2) + \Sigma_t(3) + \ldots + \Sigma_t(N)}$$

This is the II. additivity of the cross sections for composite materials

Note that this additivity is NOT valid for the microscopic cross sections!

The mean free path is the average distance that a particle travels in the matter without any interaction (reaction)

We saw: $\phi(x) = \phi_0 e^{-\Sigma_t \cdot x}$ This shows what fraction of the initial flux reached the x depth without any interaction.

The probability density function for this is $p(x) \propto \frac{\phi(x)}{\phi_0} = e^{-\Sigma_t \cdot x}$

The expectation value of the distance travelled without interaction:

$$\langle x \rangle = \frac{\int x \cdot p(x) dx}{\int p(x) dx} = \frac{1}{\Sigma_t}$$

The mean free path is: $\boxed{\Lambda = \frac{1}{\Sigma_t}}$

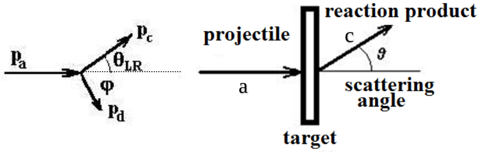
Differential cross section

Definition: $\boxed{\sigma = \frac{R}{N \cdot \phi}} \longleftarrow$ The rate of what we are „curious" about

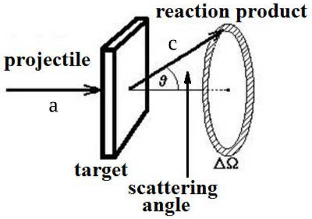
We could be curious about some details as well!

Angular distributio

a + b ⟶ c + d



In 3D we want to know how many particles fly in the $d\Omega$ solid angle determined by the $(\vartheta, \vartheta + d\vartheta)$ angular region, for given N and ϕ.



The solid angle between $(\vartheta, \vartheta + d\vartheta)$: $d\Omega = 2\pi \cdot \sin\vartheta \cdot d\vartheta$ (with cylindrical symmetry)

Differential cross section: a cross section „exploded" according to certain parameter

Exploded according to solid angle: $\boxed{\frac{d\sigma}{d\Omega} = f(\vartheta)} \longleftarrow$ Depends on the scattering angle

Its unit is: barn/steradian

Obviously, integrating for the scattering angles

$$\int_0^\pi \frac{d\sigma}{d\Omega} \cdot 2\pi \cdot \sin\vartheta \cdot d\vartheta = \sigma$$

We get back the cross section

Sometimes we do not explode it according to the solid angle, but according to the scattering angle:

$$\frac{d\sigma}{d\vartheta} = 2\pi \cdot \sin\vartheta \cdot \frac{d\sigma}{d\Omega}$$

, as it can easily be demonstrated.

Energy dependence of the cross sections (excitation functions)

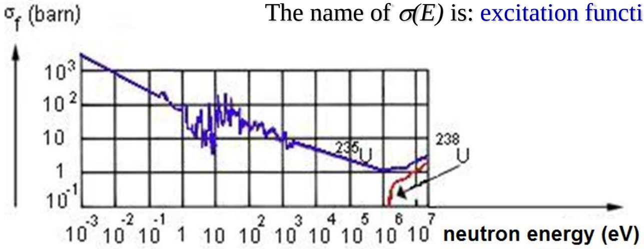
Cross section: $\boxed{\sigma(E) = \frac{R(E)}{N \cdot \left[\frac{\mathrm{d}\phi}{\mathrm{d}E}(E) \cdot \mathrm{d}E\right]}}$

There can be particles with different energy in the flux.

⟵ The flux in the interval (E, E+dE)

For example: neutrons with different energy cause reactions with different probabilities, therefore the cross section will be energy dependent.

The name of $\sigma(E)$ is: excitation function



^{235}U and ^{238}U fission cross section depends on the neutron energy

Nuclear reaction mechanisms

Potential scattering

Direct reactions

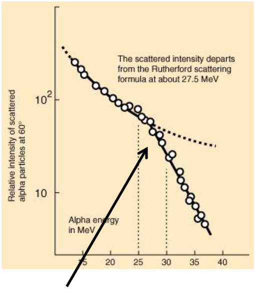
Compound nucleus forming nuclear reactions

These are „extreme" models, of course.

1. Potential scattering

The projectile interacts (gets scattered) only by the (nuclear) potential of the target nucleus, the internal structure of the nucleus does not change, no exchange of nucleons etc. occurs. „Optical" model, „unclear glass sphere model"→ the particles scatter on the nucleus as light scatters on an unclear glass sphere („unclear" glass describes some absorption)

It usually occurs at small projectile energies.



It „feels" also the nuclear interaction

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2. Direct reactions

The interaction between the projectile and the target (or with a part of the target nucleus) occurs fast, in one step.

What does „fast" mean? As compared to what?

Example: consider protons with 10 MeV energy

$$\frac{1}{2}mv^2 = 10\ \mathrm{MeV} = 1{,}6\cdot 10^{-12}\ \mathrm{J}$$ We get for the velocity

$$v = \sqrt{\frac{3{,}2\cdot 10^{-12}}{1{,}67\cdot 10^{-27}}} = 4{,}4\cdot 10^{7}\ \frac{\mathrm{m}}{\mathrm{s}}$$ The size of a nucleus is $R \sim 10^{-14}$ m,

The „interaction time" between the protons and the nucleus:

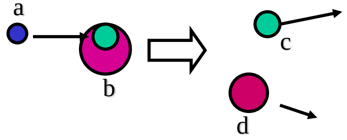
$$t = \frac{2R}{v} \approx 8{,}8\cdot 10^{-21}\ \mathrm{s} \sim 10^{-20}\ \mathrm{s}$$

This is the order of magnitude of the time of the direct reactions.

At the direct reactions the projectile interacts only with one or a few nucleon of the nucleus. The rest of the nucleus is not involved in the reaction, we call it „spectator".

Most important types of direct reactions

a) Knock-out reaction

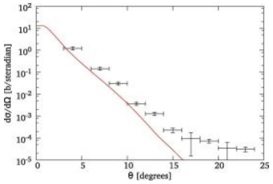


The projectile interacts with a nucleon (or with a small cluster or nucleons), and knocks it out of the nucleus

Typical reactions: high energy projectiles,
(n,n'), (n,p), (p,n), (p,p'), (α,n), (α,p) etc.

Characteristics:

- the knocked out particle is emitted in „forward" direction, i.e. the differential cross section is large at small angles and small at large angles („forward scattering")
- the knocked out particle gets a substantial part of the total momentum, the rest of the nucleus is only slightly kicked.



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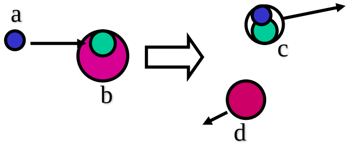
b) Pick-up reaction

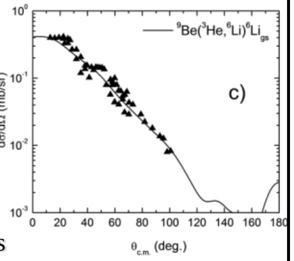


The projectile picks up a nucleon (or a small cluster of nucleons), and they leave together

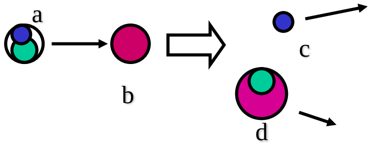
Typical reactions: high energy projectiles,
(n,d), (p,d), (d,^{6}Li), (^{3}He,^{6}Li), (α,^{6}Li) etc.

Characteristics:

- the resulting particle is emitted in „forward" direction, i.e. the differential cross section is large at small angles and small at large angles („forward scattering").
- the velocity of the emitted particle ~ velocity of the projectile, therefore the momentum of the emitted particle is larger than that of the projectile. The remnant gets a small „backward" momentum.



c) Stripping reaction



The target nucleus „strips off” a nucleon (or small cluster of nucleons) from the projectile, and only the remaining part will be emitted

Typical reactions: high energy projectiles,
(d, n), (d, p), (^{6}Li, d), (^{6}Li, α) etc.

Characteristics:

- the remaining particle is emitted in „forward” direction, i.e. the differential cross section is large at small angles and small at large angles („forward scattering”).
- the velocity of the remaining particle is about the same as was the velocity of the projectile, therefore its momentum is smaller
- the target nucleus gets the momentum, which the stripped-off part of the projectile had before the reaction.

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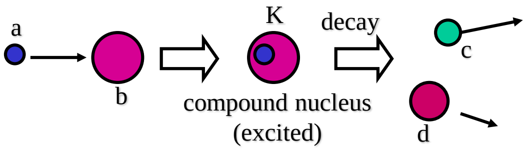
3. Nuclear reactions with „compound nucleus” mechanism

We assume that the reaction occurs in two consecutive steps:

a) The projectile fusions with the target nucleus, a new nucleus is formed: this is the compound nucleus (or intermediate nucleus). The reaction energy of the fusion will be distributed to all degrees of freedom – „thermalization”. The compound nucleus is created in an excited state.

b) The excited compound nucleus decays into a decay „channel”



Characteristics of the compound nucleus reaction mechanism:

a) The time of the reaction is much larger than for the direct reactions (t > 10^{-16} s).

b) The compound nucleus has a level scheme, and it can be formed only in one of the allowed levels → „resonances” in the formation cross-section!

c) Because the reaction energy will be distributed to all degrees of freedom, a „thermal equilibrium” (thermalization) occurs. Therefore the compound nucleus does not „remember” how it was formed. This has several consequences:

α) The angular distribution of the particles emitted during the decay is not depending on the direction of the projectile (isotropic angular distribution in CM system)

β) The decay mode is determined only by the excited state of the compound nucleus (not depending on the mode of the formation of the compound nucleus). Branching ratios

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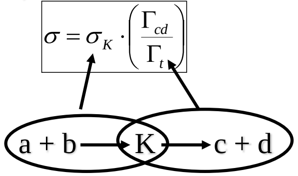
d) The cross section of the reaction can be split into to factors:



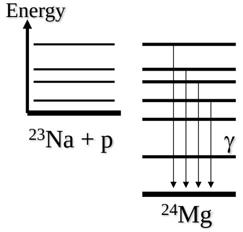
$$\boxed{\sigma = \sigma_K \cdot \left(\frac{\Gamma_{cd}}{\Gamma_t}\right)}$$

Here σ_K is the cross section for the formation of the compound nucleus

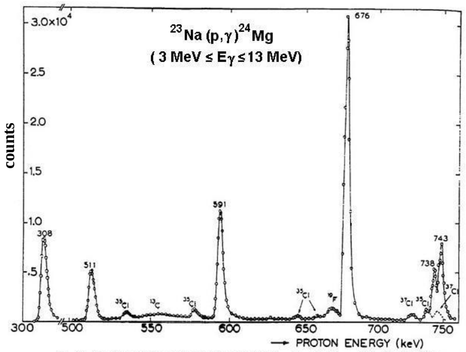
$\left(\frac{\Gamma_{cd}}{\Gamma_t}\right)$ is the „branching ratio”.

The branching ratio shows the ratio of the decay into the particular c + d channel from all possible decay modes.

1. Example The formation of the compound nucleus is possible on at certain energies ⟶ resonances



Experiment: $^{23}Na(p,\gamma)^{24}Mg$



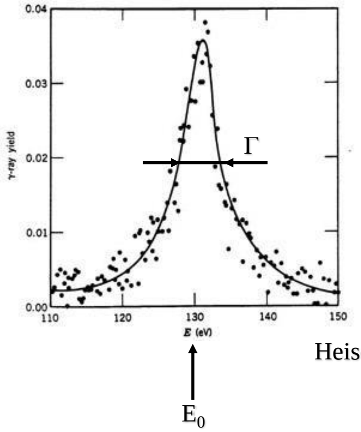
P. W. M. Glaudemans and P. M. Endt, *Nucl. Phys.* **30**, 30 (1962).

The shape of the resonances

Breit-Wigner formula:

$$\boxed{\sigma(E) \sim \frac{\left(\Gamma/2\right)^2}{\left(E-E_0\right)^2+\left(\Gamma/2\right)^2}}$$

„place” „width”



The width of the resonance and the τ lifetime of the state are related:

$$\boxed{\Gamma \cdot \tau \approx \frac{h}{2\pi}}$$

and with the T half life:

$$\boxed{\Gamma \cdot T \approx \frac{h}{2\pi} \cdot \ln 2}$$

here h is the Planck-constant

Heisenberg uncertainty principles
(quantum mechanics)

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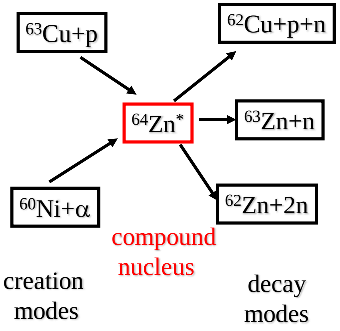
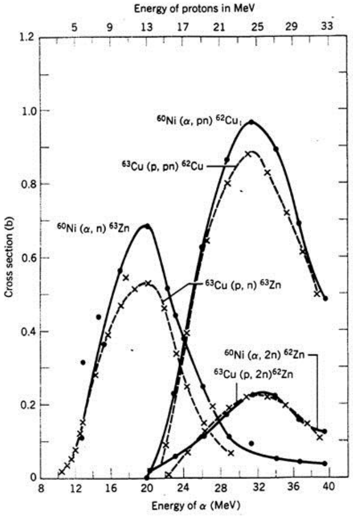
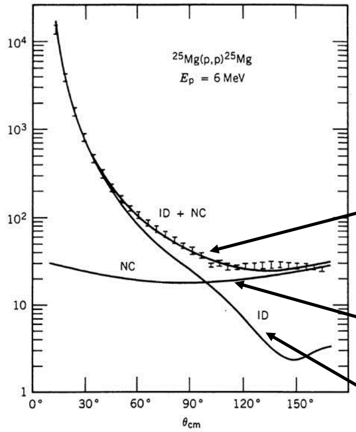
2. Example:
The decay mode does not depend on the formation





There are cases, when the „direct” and „compound” reaction mechanisms are „mixed”.

For example:
angular distribution of
$^{25}Mg(p,p')^{25}Mg$



Experimental data

„compound” contribution
(almost isotropic)

„direct” contribution
(forward scattering)