Nuclear Physics
(11^{th} lecture)

Content

THE NUCLEAR INTERACTION

- **The deuteron**
- **Yukawa model for the nuclear interaction**
- **Spin dependency and tensor force**
- **Charge independency and isospin**
- **Mirror nuclei, isobars and isobar analogue states**

1

Nuclear interaction

1) The deuteron

The simplest nucleus, where the nuclear forces can be studied

Bound system of a proton and a neutron

Unfortunately there are only a few experimental facts that can be studied, since it exists only in ground state (no excited states, no energy level scheme)

Experimental facts:

- **Binding energy: 2,2 MeV**
- **Angular momentum:** $J = 1\hbar$
- **Magnetic dipole momentum:** $\mu = 0{,}857411 \pm 0{,}000019\ \mu_N$
- **Electric quadrupole momentum:** $Q = +2{,}74\ \text{mb}$

What can we learn about the nuclear interaction from these facts ?

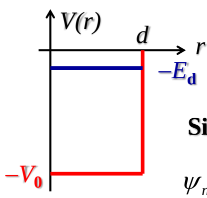
2

The deuteron (contd.)

- **First, since the two nucleons are bound together, the nuclear interaction must be attractive**
- **From the success of the liquid drop model we know that the nuclear interaction must be short ranged**
- **Assume that it depends only on the absolute value of the distance (central):** $V(r)$.

1.) The binding energy of the deuteron

The simplest short range attractive potential is the square well:



$$V(r) = \begin{cases} -V_0 & \text{if } r \le d \\ 0 & \text{if } r > d \end{cases}$$

Since the potential is central, the wave function:

$$\psi_{n_r,l,m}(r,\vartheta,\varphi) = \frac{\mathscr{R}_{n_r}(r)}{r} Y_l^m(\vartheta,\varphi)$$

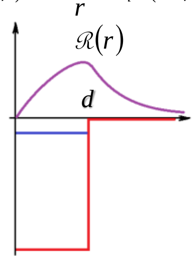
3

The deuteron (contd.)

$$\psi_{n_r,l,m}(r,\vartheta,\varphi) = \frac{\mathscr{R}_{n_r}(r)}{r} Y_l^m(\vartheta,\varphi)$$

In the ground state we assume $l = 0$

The Schrödinger equation is easy to solve for the square well potential:



$$\mathscr{R}_0(r) = \mathscr{R}(r) = \begin{cases} A\sin kr & \text{if } r < d \\ Be^{-\kappa r} & \text{if } r \ge d \end{cases}$$

where $k = \sqrt{\frac{M}{\hbar^2}(V_0 - E_d)}$ **and** $\kappa = \sqrt{\frac{M}{\hbar^2}E_d}$

The solutions inside and outside the potential should be joined, therefore the boundary condition: $\left(\frac{1}{\mathscr{R}} \cdot \frac{\mathrm{d}\mathscr{R}}{\mathrm{d}r}\right)_{d-\varepsilon} = \left(\frac{1}{\mathscr{R}} \cdot \frac{\mathrm{d}\mathscr{R}}{\mathrm{d}r}\right)_{d+\varepsilon}$

We get $k \cdot \operatorname{cotan}(d \cdot k) = -\kappa$

Finally: $\operatorname{cotan}\frac{d}{\hbar}\sqrt{M(V_0 - E_d)} = -\sqrt{\frac{E_d}{V_0 - E_d}}$

4

The deuteron (contd.)

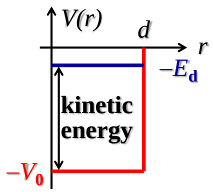
$$\operatorname{cotan}\frac{d}{\hbar}\sqrt{M(V_0-E_d)}=-\sqrt{\frac{E_d}{V_0-E_d}}$$

Simplification: what if the deuteron would be „almost" unbound, i.e. $E_d = 0$? $\Rightarrow$ $\operatorname{cotan}\frac{d}{\hbar}\sqrt{MV_0'}=0$ $\Rightarrow$ $\frac{d}{\hbar}\sqrt{MV_0'}=\frac{\pi}{2}$

We get $V_0'=\frac{\pi^2\hbar^2}{4M}\cdot\frac{1}{d^2}=\frac{102}{d^2}[\mathrm{MeV}]$, **if d is in fm.**

From $R=r_0\sqrt[3]{A}$ **we know that the range is smaller than 2 fm, so** $V_0'>25{,}5\,\mathrm{MeV}$. **Obviously** $V_0>V_0'$

This means that the deuteron is only weakly bound! $E_d \ll V_0$



Calculation shows that the expectation value of the distance between the proton and the neutron is larger than d (!)

5

The deuteron (contd.)

2.) The spin of the deuteron

$\mathbf{J}=\mathbf{L}+\mathbf{S}$

We assumed L= 0, so the angular momentum comes from S.

$$\mathbf{S}=\mathbf{S}_p+\mathbf{S}_n \Rightarrow S=\begin{cases}1\,(\text{triplet}) & \uparrow\uparrow\\ 0\,(\text{singlet}) & \uparrow\downarrow\end{cases}$$

If the nuclear interaction was independent of spin alignment, then the 4 cases would be equivalent $\Rightarrow$ ¾ of the deuterons had J = 1, ¼ of the deuterons had J = 0.

Experimental fact: $J=1\hbar$ **always!**

$\Downarrow$

The $V(r)$ nuclear interaction should be spin-dependent so, that the triplet case is favoured!

6

The deuteron (contd.)

3.) The magnetic momentum of the deuteron

We assumed L= 0, so no magnetic momentum can arise from orbital motion. $\Rightarrow$ $\mu=\mu_p+\mu_n$ **(since the spins are parallel)**

$$\mu_p=(2{,}79275\pm0{,}00003)\mu_N$$
$$\mu_n=(-1{,}91315\pm0{,}00007)\mu_N$$
$$\Rightarrow \mu_p+\mu_n=(0{,}87960\pm0{,}0001)\mu_N$$

Experimental fact: $\mu_d=(0{,}857411\pm0{,}00002)\mu_N$

„Almost" good, but not completely!
This is the anomalous magnetic moment of the deuteron.

What can be the reason? Maybe $L\neq 0$?

7

The deuteron (contd.)

Maybe $\mathbf{L}\neq 0$?

If there is an L, then the operator of the magnetic momentum:

$$\mathbf{M}=\mu_N\frac{1}{2}\left[\left(\frac{1}{2}+g_{sn}+g_{sp}\right)(\mathbf{L}+\mathbf{S})+\left(\frac{1}{2}-g_{sn}-g_{sp}\right)(\mathbf{L}-\mathbf{S})+2(g_{sn}-g_{sp})(\mathbf{S}_n-\mathbf{S}_p)\right]$$

Where $\mathbf{S}=\mathbf{S}_n+\mathbf{S}_p$, **and** $g_{sp}=5{,}586$ **and** $g_{sn}=-3{,}826$.

The projection of the magnetic momentum on the angular momentum: $\mu=\left\langle\psi_{L,S,J,m_j=J}\middle|M_3\middle|\psi_{L,S,J,m_j=J}\right\rangle$

Evaluation is tedious but straightforward, and we get for

$$\boldsymbol{\mu}=g_j\mathbf{J}\mu_N$$

$$g_j=\frac{1}{2}\left[\left(\frac{1}{2}+g_{sn}+g_{sp}\right)+\left(\frac{1}{2}-g_{sn}-g_{sp}\right)\frac{L(L+1)-S(S+1)}{J(J+1)}\right]$$

8

The deuteron (contd.)

We know that $J = 1$, and $\mathbf{J} = \mathbf{L} + \mathbf{S}$

Possibilities

L	Spin	π	g_j	g_j
$L = 0$	$S = 1$	+	$g_{sn} + g_{sp}$	1,7592
$L = 1$	$S = 1$	–	$1/4+(g_{sn}+g_{sp})/2$	1,1296
$L = 1$	$S = 0$	–	1/2	0,5
$L = 2$	$S = 1$	+	$3/4-(g_{sn}+g_{sp})/2$	-0,1296

Measured value: $\boxed{g_j = 1{,}71482}$

No single orbit can explain the measured value!

Superposition (mixture)? $\psi = a\psi_a + b\psi_b$ where $|a|^2 + |b|^2 = 1$

$g_j = |a|^2 g_{ja} + \left(1-|a|^2\right) g_{jb}$ This means that $g_{ja} \le g_j \le g_{jb}$

Obviously, the $L = 0$, $S = 1$ case should be included!

Since parity should be a good quantum number, the only possible contribution is $L = 2$, $S = 1$

9

The deuteron (contd.)

Substituting the appropriate values and solving the

$g_j = |a|^2 g_{ja} + \left(1-|a|^2\right) g_{jb}$ equation

we get $|a|^2 = 0{,}9765$ and $|b|^2 = 0{,}0235$

Only ~2% of the $L = 2$ $S = 1$ wave-function should be mixed!

What does L-mixing mean for the properties of the nuclear interaction?

We know: for central potentials L is a good quantum number.

If L-mixing occurs, then L is not a good quantum number
→ the potential cannot be central!

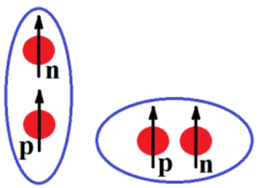
10

The deuteron (contd.)

4.) The quadrupole momentum of the deuteron

Experimental fact:

Electric quadrupole momentum: $Q = +2{,}74 \text{ mb} > 0$



$Q > 0$ $Q < 0$

Conclusion: the $V(\mathbf{r})$ potential should depend on the $(\sigma_1\mathbf{r})$ and $(\sigma_2\mathbf{r})$ scalar products (on the angles between the spins and the distance vector)

Analogy: the interaction between two magnetic dipoles (electrons):

$$V = -\left(\frac{e\hbar}{2mc}\right)\cdot\frac{1}{r^3}\left[\frac{3(\boldsymbol{\sigma}_1\mathbf{r})(\boldsymbol{\sigma}_2\mathbf{r})}{r^2} - \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\right]$$

For the nucleons: it is NOT of electromagnetic origin, but similar shape:

$$V = V_T(r)\left[\frac{3(\boldsymbol{\sigma}_n\mathbf{r})(\boldsymbol{\sigma}_p\mathbf{r})}{r^2} - \boldsymbol{\sigma}_n\boldsymbol{\sigma}_p\right] = V_T(r)\cdot \mathbf{S}_{n,p}$$

Tensor force

11

The deuteron (contd.) $V = V_T(r)\cdot\left[\frac{3(\boldsymbol{\sigma}_n\mathbf{r})(\boldsymbol{\sigma}_p\mathbf{r})}{r^2} - \boldsymbol{\sigma}_n\boldsymbol{\sigma}_p\right] = V_T(r)\cdot \mathbf{S}_{n,p}$

If $V_T(r)$ is attractive, then this type of interaction gives explanation for

- the sign of the quadrupole momentum (cigar shape)
- the anomaly of the magnetic momentum (it is not central)
- the spin-dependency of the two-nucleon interaction
  (for $S = 1$ it is attractive, for $S = 0$ repulsive)

Summarized:
The nucleon-nucleon interaction is
- Strong,
- Attractive,
- Short range,
- Spin-dependent,
- Non-central,
- Tensor force,
- Charge independent (see later)

12

Yukawa nuclear interaction

Hideki Yukawa
(1907-1981)
Nobel-prize 1949

Explains the short range by
- using the field-theory approach, in analogy with the quantum-electrodynamics (was then a new theory)
- assuming a massive force-carrier boson

We think now that the different types of interactions between particles are carried by (emitting and adsorbing) force-carrier bosons

Interaction	Carrier	Mass (GeV/c²)	Spin
Strong (between quarks)	8 gluons	~0	1
Electromagnetic	photon	0	1
Weak	W^+, W^-, Z^0	80, 80, 91	1
Gravity	graviton	0	2

These force-carriers are virtual particles (see later)

Yukawa nuclear interaction (contd.)

Particles usually have nonzero energy.
Massive particles have energy even at rest: mc^2.

How is it possible to „create" the carrier particles?
Where does the energy come from?

Heisenberg-relation for energy and time: $\Delta E \cdot \Delta t \approx \hbar$

Interpretation: the vacuum can give a „loan" of ΔE energy, but only for a limited Δt time!

Assume that we want to create a carrier with $\Delta E = mc^2$.

Then the maximal time of the „loan" is $\Delta t \approx \frac{\hbar}{mc^2}$

The carrier cannot travel faster than the light, so for the range of the interaction : $d \le c \cdot \Delta t \approx \frac{\hbar}{mc}$

The higher is the mass of the carrier, the shorter is the interaction range

Yukawa nuclear interaction (contd.)

$d = \frac{\hbar}{mc}$

Yukawa: the range of the nucleon-nucleon interaction is ~ 1 fm.
The m of the carrier can be calculated: $m \sim 250\ m_e$. (meson)
Discovery of the pion (π-meson) in 1947: $m_\pi \sim 135$ MeV/c² (270 m_e)
(Cecil Frank Powell et al, Nobel-prize in 1950)

Quantum mechanics $E^2 = -\hbar^2 \frac{\partial^2}{\partial t^2}$ and $p^2 = -\hbar^2 \nabla^2$

electromagnetic interaction

$$E^2 = p^2c^2 \Rightarrow p^2 - \frac{1}{c^2}E^2 = 0$$

nucleon-nucleon interaction

$$E^2 = p^2c^2 + (mc^2)^2 \Rightarrow p^2 + m^2c^2 - \frac{1}{c^2}E^2 = 0$$

Φ is the potential

$$-\hbar^2\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0$$

$$-\hbar^2\left(\nabla^2 - \left(\frac{mc}{\hbar}\right)^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = 0$$

For point-like source, stationary case

$$\nabla^2\phi = -\frac{e}{\varepsilon_0}\delta(\mathbf{r}-\mathbf{r}_0)$$

$$\phi = \frac{e}{4\pi\varepsilon_0}\cdot\frac{1}{|\mathbf{r}-\mathbf{r}_0|}$$

„mesonic charge"

$$\left(\nabla^2 - \left(\frac{mc}{\hbar}\right)^2\right)\phi = 4\pi g\,\delta(\mathbf{r}-\mathbf{r}_0)$$

$$\phi = -g\cdot\frac{e^{-\frac{mc}{\hbar}|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}$$

Yukawa nuclear interaction (contd.)

$$\phi = -g\cdot\frac{e^{-\frac{mc}{\hbar}|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}$$

Denote: $\mathbf{r}-\mathbf{r}_0 \to \mathbf{r}$ (put the origin into point $\mathbf{r}_0$).
From this the interaction energy:

$$\boxed{V(\mathbf{r}) = g\cdot\phi = -g^2\cdot\frac{e^{-\frac{mc}{\hbar}r}}{r}}$$

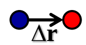
This is the Yukawa interaction in its simplest form.
(Compare it to the „range" $d \approx \frac{\hbar}{mc}$)

However, this is a „central" interaction! How to make is spin-dependent and tensorial?

Assume that the source of the field is not point-like, but dipole-like meson field!

$$D(\mathbf{r}) = -4\pi g \lim_{\Delta r\to 0}\left(\delta(\mathbf{r}+\Delta\mathbf{r}) - \delta(\mathbf{r})\right) \approx -4\pi\cdot(g\Delta\mathbf{r})\cdot\nabla\delta(\mathbf{r})$$

$\mathbf{M} = g\cdot\Delta\mathbf{r}$ – „mesonic dipole charge"



With that: $\left(\nabla^2 - \left(\frac{mc}{\hbar}\right)^2\right)\phi = -4\pi\mathbf{M}\nabla\delta(\mathbf{r}) \Rightarrow \phi = \mathbf{M}\nabla\cdot\frac{e^{-\frac{mc}{\hbar}r}}{r}$

Relate the direction of the nuclear „mesonic dipole charge" to the spin, as the only special direction: $\mathbf{M} = f\cdot\mathbf{s}$

Yukawa nuclear interaction (contd.)

$$\boxed{\begin{aligned}V(\mathbf{r},\mathbf{s}_1,\mathbf{s}_2)&=(\mathbf{M}_1\nabla)(\mathbf{M}_2\nabla)\cdot\frac{e^{-\frac{mc}{\hbar}r}}{r}=\\&=f^2\left[\frac{1}{3}\mathbf{s}_1\mathbf{s}_2\left(\left(\frac{mc}{\hbar}\right)^2\frac{e^{-\frac{mc}{\hbar}r}}{r}-4\pi\delta(r)\right)+S_{1,2}\left(\frac{1}{r^2}+\frac{1}{r}\cdot\frac{mc}{\hbar}+\frac{1}{3}\left(\frac{mc}{\hbar}\right)^3\right)\frac{e^{-\frac{mc}{\hbar}r}}{r}\right]\end{aligned}}$$

where $S_{1,2}=\frac{3(\mathbf{s}_1\mathbf{r})(\mathbf{s}_2\mathbf{r})}{r^2}-\mathbf{s}_1\mathbf{s}_2$ **is the tensor interaction**

Looks good, since it has

- **short range** ☺
- **spin dependency** ☺
- **tensor force** ☺

Calculate for the deuteron:

$\mathbf{S}=\mathbf{S}_1+\mathbf{S}_2 \qquad (\mathbf{S}_i\cdot\mathbf{r})=S_i\cdot r$

$(\mathbf{S}_1\mathbf{S}_2)=\frac{1}{2}\left(S^2-S_1^2-S_2^2\right)$

$\langle(\mathbf{S}_1\mathbf{S}_2)\rangle=\frac{1}{2}\left(S(S+1)-S_1(S_1+1)-S_2(S_2+1)\right)$

$\langle(\mathbf{S}_1\mathbf{S}_2)\rangle=\frac{1}{2}\left(2-\frac{3}{4}-\frac{3}{4}\right)=\frac{1}{4}>0$

Repulsion, not attraction! ☹

Nuclear interaction

Isospin and charge independency

There is another property of the nucleon-nucleon interaction which was not considered yet: charge independency!

Charge independency means: the nuclear interactions is the same for $p-p$, $n-n$ **and** $n-p$ **systems.**

Heisenberg/Wigner: the proton and the neutron are the same particle (nucleon), only an „internal" two-state quantum-number is different. (Like e↑ and e↓ are both electrons, only the spin z **- projection quantum number is opposite).**

$\psi(\mathbf{r})=\varphi(\mathbf{r})\cdot\alpha$, **where** $\alpha=\pi=\begin{pmatrix}1\\0\end{pmatrix}$ **for protons and** $\alpha=\nu=\begin{pmatrix}0\\1\end{pmatrix}$ **for neutrons.**

This internal quantum number is called „isospin".
(It has nothing to do with the „normal spin", and with the angular momentum, even it is not in the configuration space)

Isospin and charge independency (contd.)

The isospin (vector) operator is denoted by T.
It has 3 components: T_1, T_2, T_3 **(NOT** T_x, T_y, T_z **!!)**

The isospin algebra is the same as the spin-algebra, although the isospin acts in an abstract space.

The commutation relations: $\left[\hat{T}_i,\hat{T}_j\right]=i\varepsilon_{ijk}\hat{T}_k \quad \left[\hat{T}^2,\hat{T}_3\right]=0$

The eigenvalues: $\hat{T}^2\to T(T+1)$ **and** $\hat{T}_3\to m_T$, where $-T\le m_T\le +T$

The ladder (step) operators: $\begin{matrix}\hat{T}^+\pi=0, & \hat{T}^+\nu=\pi\\ \hat{T}^-\pi=\nu, & \hat{T}^-\nu=0\end{matrix}$ **where** $\begin{matrix}\hat{T}^+=\hat{T}_1+i\hat{T}_2\\ \hat{T}^-=\hat{T}_1-i\hat{T}_2\end{matrix}$

For nucleons: $T=\frac{1}{2}$

For protons $\pi=\begin{pmatrix}1\\0\end{pmatrix}$ **and neutrons** $\nu=\begin{pmatrix}0\\1\end{pmatrix}$**:** $\boxed{\hat{T}_3\pi=\frac{1}{2}\pi}$ $\boxed{\hat{T}_3\nu=-\frac{1}{2}\nu}$

Consequence of the new quantum number:
The Pauli-principle → the whole wave function should be antisymmetric, including the new quantum number!

Isospin and charge independency (contd.)

How to handle mathematically the exchange of particles?
Suppose that originally the „1" particle is a proton, and the „2" is a neutron $\psi(\mathbf{r},s_1,s_2,t_1,t_2)=\varphi(\mathbf{r},s_1,s_2)\cdot\pi_1\cdot\nu_2$
(for simplicity we will omit the φ **(r,** s_1, s_2**) part in the following)**
For the exchange we have to „turn" the first proton to neutron, and the second neutron into a proton : $\hat{T}^-(1)\hat{T}^+(2)\pi_1\nu_2=\nu_1\pi_2$

However, this operator turns a $\nu_1\pi_2$ **system into 0, thus this operator should be expanded to enable the exchange for both systems:**

$\left(\hat{T}^+(1)\hat{T}^-(2)+\hat{T}^-(1)\hat{T}^+(2)\right)\pi_1\nu_2=\nu_1\pi_2$

$\left(\hat{T}^+(1)\hat{T}^-(2)+\hat{T}^-(1)\hat{T}^+(2)\right)\nu_1\pi_2=\pi_1\nu_2$

Note that $\hat{T}^+(1)\hat{T}^-(2)+\hat{T}^-(1)\hat{T}^+(2)\propto\hat{T}_1(1)\hat{T}_1(2)+\hat{T}_2(1)\hat{T}_2(2)$

Note also that this operator gives 0 for p-p and n-n systems.
To ensure the effect also on these symmetrical states, $\hat{T}_3(1)\hat{T}_3(2)$ **should be included. Finally we get:**

$\boxed{\hat{T}_1(1)\hat{T}_1(2)+\hat{T}_2(1)\hat{T}_2(2)+\hat{T}_3(1)\hat{T}_3(2)\equiv\hat{\mathbf{T}}(1)\hat{\mathbf{T}}(2)}$ **(scalar product)**

Isospin and charge independency (contd.)

Therefore the potential with charge-independency:

$$\boxed{\begin{aligned}&V(\mathbf{r},\mathbf{s}_1,\mathbf{s}_2,t_1,t_2)=\\&= f^2\mathbf{T}_1\mathbf{T}_2\left[\frac{1}{3}\mathbf{s}_1\mathbf{s}_2\left(\left(\frac{mc}{\hbar}\right)^2\frac{e^{-\frac{mc}{\hbar}r}}{r}-4\pi\delta(r)\right)+S_{1,2}\left(\frac{1}{r^2}+\frac{1}{r}\cdot\frac{mc}{\hbar}+\frac{1}{3}\left(\frac{mc}{\hbar}\right)^3\right)\frac{e^{-\frac{mc}{\hbar}r}}{r}\right]\end{aligned}}$$

The isospin of the deuteron: $\mathbf{T}=\mathbf{T}_1+\mathbf{T}_2$**, therefore** $\mathbf{T}_1\mathbf{T}_2=\frac{1}{2}\left(T^2-T_1^2-T_2^2\right)$

The expectation value: $\langle\mathbf{T}_1\mathbf{T}_2\rangle=\frac{1}{2}\left(T(T+1)-\frac{3}{4}-\frac{3}{4}\right)=\frac{1}{4}(2T(T+1)-3)$

Two possibilities: $T=\begin{cases}0 & M_T=0\\ 1 & M_T=0,\pm1\end{cases}$

For the deuteron $S = 1$, $L = 0, 2$ → symmetrical against particle exchange → the isospin must be antisymmetric → $T = 0$

$$\psi(\mathbf{r},s_1,s_2,t_1,t_2)=\varphi(\mathbf{r},s_1,s_2)\frac{1}{\sqrt{2}}(\pi_1\nu_2-\nu_1\pi_2)$$

21

Isospin and charge independency (contd.)

Physical meaning of the isospin: $\hat{T}_3\psi=\sum_{i=1}^{A}\hat{T}_3(i)\psi=\frac{1}{2}(Z-N)\psi$

Since every proton gives +1/2, every neutron gives –1/2.

The Hamiltonian of the nucleus: $H = H_0 + H_a$ **, where** $[H_0,T_i]=0$

$H_0=H_{kin}+H_{Yukawa}+A\frac{M_n+M_p}{2}c^2$ **since** $M_n\neq M_p$

$H_a=H_{Coulomb}+N\frac{M_n-M_p}{2}c^2+Z\frac{M_p-M_n}{2}c^2$ **Clearly** $[H_a,T_i]\neq0$

If H_a was not there, the isobars would have exactly the same structure, since exchange of protons and neutrons would not influence anything!

22

Mirror nuclei

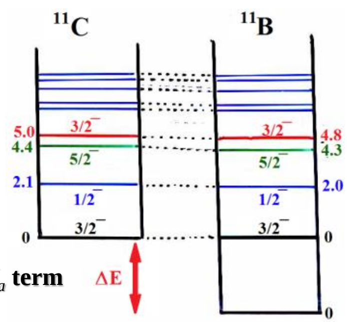
For mirror nuclei number of protons and neutrons are interchanged

For example: ${}^{11}_{5}\mathrm{B}_6$ **and** ${}^{11}_{6}\mathrm{C}_5$

${}^{11}_{5}\mathrm{B}_6 \ \Rightarrow T_3=\frac{1}{2}(5-6)=-\frac{1}{2}$

${}^{11}_{6}\mathrm{C}_5 \ \Rightarrow T_3=\frac{1}{2}(6-5)=+\frac{1}{2}$

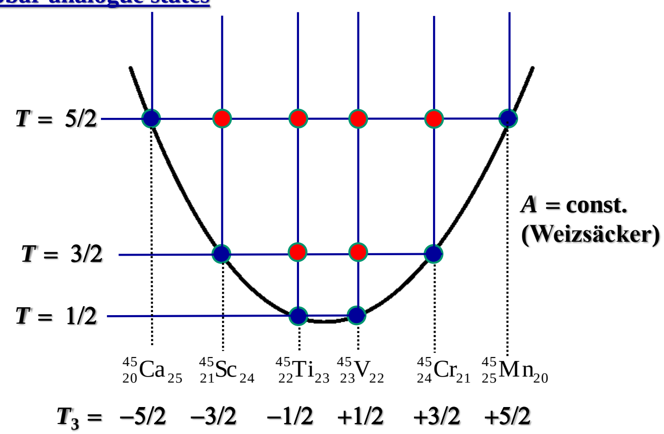
They are $T = 1/2$ isospin doublet



The origin of ΔE is the H_a term

23

Isobar analogue states



Isobar analogue states → they have the same T → their internal structure is similar

24