Nuclear Physics
(10th lecture)

Content

- Nuclear Collective Model: Rainwater approx. (reminder)
- Consequences of nuclear deformation
  - Rotational states
    - High spin states and back bending
  - Vibrational states
    - Monopole, dipole and quadrupole vibrations,
    - Giant resonances, experimental observations

The nuclear collective model (contd.)

Rainwater approximation (reminder)

„Marriage" of the liquid drop model and the shell model.

Concept: nucleus = core + valence nucleon(s)

core ↑ deformable liquid drop

valence nucleon(s) ↑ shell model

Consequence #1: Some nuclei are deformed also in ground state.

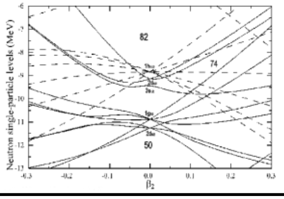
Consequence #2: Nuclear quadrupole moment:

$$Q=\left(1+\frac{4}{5}\frac{ZR^2}{\langle r^2\rangle}\cdot\frac{U_0}{S}\right)Q_N$$

Here S is the „rigidity" of the core, and Q_N is the quadrupole moment of the valence nucleon

Consequence #3: Nilsson-scheme of shell-model levels

Better description of (deformed) ground-state properties



Further consequences of the core & potential deformation

4) No more isotropy ⟶ rotational excitation is possible
5) Deformability ⟶ vibrational excitation is possible

Rotation

Classical physics → also a sphere can rotate
Quantum physics → only deformed objects can rotate

Rotator: $E=\frac{\hbar^2}{2\theta}\mathbf{I}^2$ ⟵ moment of inertia

Total angular momentum : $\mathbf{J}=\mathbf{I}+\mathbf{j}$

$\mathbf{I}$ ↑ core rotation

$\mathbf{j}$ ⟵ valence nucleon (not a rotation!)

$J^2=I^2+j^2+2(\mathbf{Ij})$

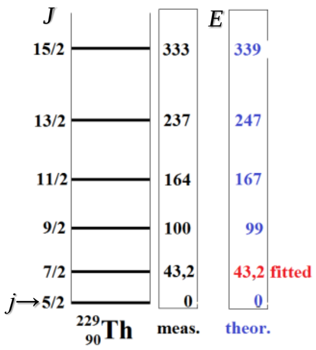
$\langle\mathbf{Ij}\rangle=0$, since I is perpendicular to the symmetry axis, and j is precessing around it.

This means that: $\langle I^2\rangle=\langle J^2\rangle-\langle j^2\rangle=J(J+1)-j(j+1)$

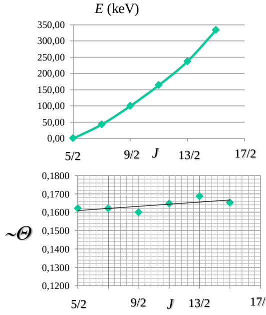
For the rotational excitations we have: $$E=\frac{\hbar^2}{2\theta}\left[J(J+1)-j(j+1)\right]$$

In the ground state I = 0, which gives $J=j$

At a rotational excitation I is increasing, so we get $J=j+1, j+2$, …



Rotational band

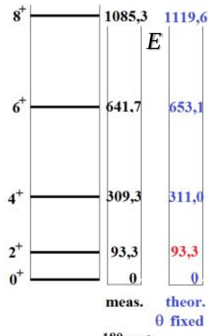


Seems that θ slightly increases (?) $\theta\propto\frac{E}{J(J+1)-j(j+1)}$

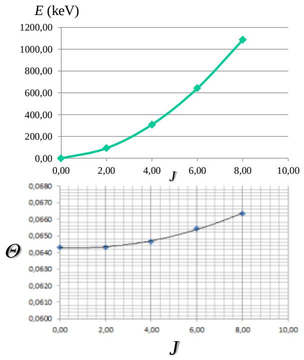
For the rotational excitations we have: $\boxed{E=\frac{\hbar^2}{2\theta}\left[J(J+1)-j(j+1)\right]}$

For even-even nuclei because of parity conservation only

J=j+2, j+4, **… can occur, and of course** ***j=0***

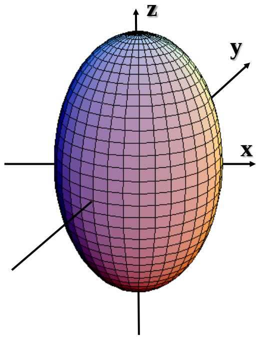


Rotational band



This way θ **can be** **measured!**
It can also be **calculated** **using the nuclear shape** $\rho(r)$
(deduced from the quadrupole moment)!

$$\theta_{theor}=\int\rho(r)r^2d^3\mathbf{r}\approx\frac{2}{5}MR^2\left(1+\frac{\varepsilon}{3}\right)$$



Big surprise: $\theta_{meas}<<\theta_{theor}$**???**

Why does it change (increases) as *J* gets larger ???

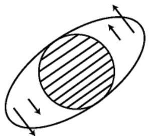
Explanation **(Aage Bohr):**
Nuclei are „superfluid" – because of the pairing (like Cooper-pairs in superconductors)

Aage Bohr
1922-2009
Nobel-prize 1975

Rotation: not like a rigid body, but more like a „surface wave"
Rigid body: $\mathbf{v}=\mathbf{r}\times\boldsymbol{\omega}\ \rightarrow\ \mathrm{rot}\,\mathbf{v}=-2\boldsymbol{\omega}$

Superfluid: no friction inside $\rightarrow\ \mathrm{rot}\,\mathbf{v}=0$
the „core" stays still, only the surface rotates

From $\mathrm{rot}\,\mathbf{v}=0$ **follows:**
$\mathbf{v}=\mathrm{grad}\,\varphi$ **(velocity potential)**



Fluid is incompressible: $\mathrm{div}\,\mathbf{v}=0$

From all these the velocity potential: $\varphi=\omega\frac{A^2-B^2}{A^2+B^2}yz$

From $\mathbf{v}=\mathrm{grad}\,\varphi$ **the velocity field can be determined.**

The angular momentum: $\mathbf{I}=\frac{M}{\hbar}\int\rho(r)[\mathbf{r}\times\mathbf{v}]d^3\mathbf{r}$

The rotational energy: $E=\frac{1}{2}M\int\rho(r)v^2d^3\mathbf{r}$

From these two $E=\frac{1}{2\theta}I^2\ \rightarrow\ \theta$ **can be calculated**

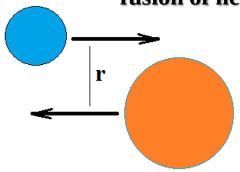
Result: $\boxed{\theta_{wave}\propto\theta_{rigid}\varepsilon^2<<\theta_{rigid}}$

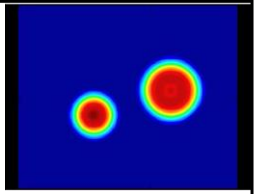
Aage Bohr, Ben Mottelson: $\boxed{\theta_{wave}\le\theta_{final}\approx\theta_{observed}\le\theta_{rigid}}$
good description, **taking into account even the pair-correlation!**

Nobel-prize 1975: A. Bohr, Mottelson, Rainwater
„…for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection."

High spin states and back bending

Producing high spin states: fusion of heavy ions





$\mathbf{I} = \mathbf{r} \times \mathbf{p}$

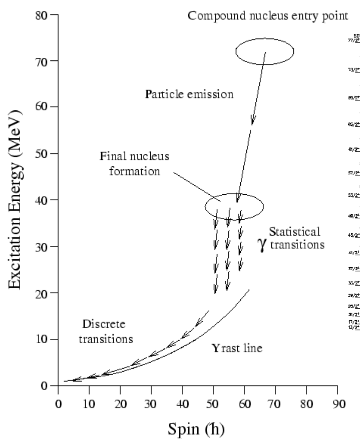
Since p is big, large angular momentum states can be formed

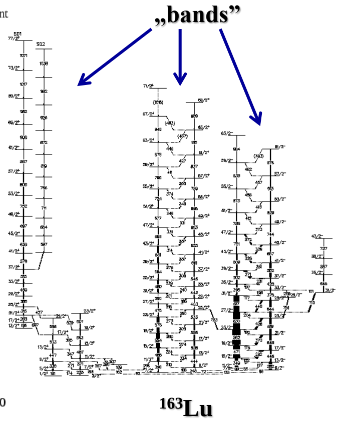
Example: 100 MeV $^{16}O + ^{222}Po \rightarrow ^{238}U$ ➡ $40-50\,\hbar$

Nuclear excitation energy: $E_{rot} = \frac{\hbar^2}{2\theta} I(I+1)$

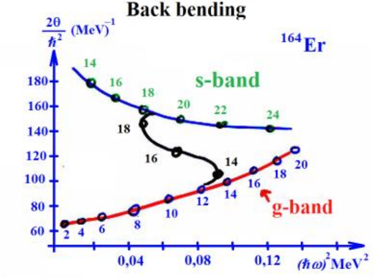
This is only the rotational energy! The nucleus may store energy in other forms (e.g. deformation, internal excitation etc.)!

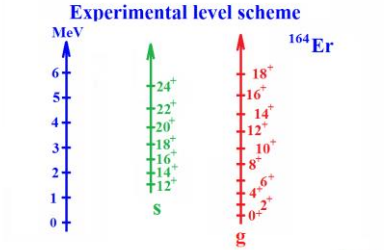
$E_x - I$ „map"





Yrast =„most rotating"





What is the reason?

Superfluid → solid transition

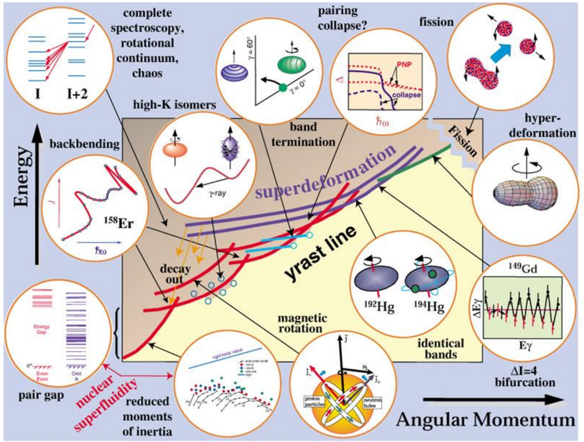
$\theta_g \rightarrow \theta_s$

What is the cause for the transition?

Coriolis-force breaks up a nucleon-pair

Rotational alignment

The picture is even more complicated ☺



http://nucalf.physics.fsu.edu/~riley/gamma/pics/hs-rev.jpg

Vibration

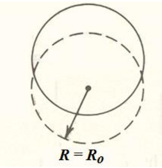
It is convenient to give the instantaneous coordinate *R(t)* of a point on the nuclear surface at *(θ, ϕ)* in terms of the spherical harmonics

$$R(t) = R_0 + \sum_{\lambda} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t)\, Y_{\lambda\mu}(\theta,\varphi)$$

Due to reflection symmetry $\alpha_{\lambda\mu} = \alpha_{\lambda,\,-\mu}$

λ=0 **vibration: monopole (GMR observed for A>40)**
(breathing mode of a compressible fluid) $E \approx 80 \cdot A^{-1/3}\ \mathrm{MeV}$

λ=1 **vibration: dipole**



$$R(t) = R_0 + \sum_{\mu=-1}^{1} \alpha_{1\mu}(t) Y_{1\mu}(\theta,\varphi) = R_0 + \alpha_{11}(t)Y_{11} + \alpha_{10}(t)Y_{10} + \alpha_{1,\,-1}(t)Y_{1,\,-1}$$

$$= R_0 + \alpha_{10}(t)Y_{10} = R_0 + \alpha_{10}(t)\ \frac{1}{2}\left(\frac{3}{2\pi}\right)^{3/2} \cos\theta$$

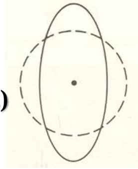
because $\alpha_{1\mu}(t) = 0$ for $\mu \neq 0$ ($\alpha_{1,\,-1}(t) = \alpha_{11}(t)$ and $Y_{1,\,-1} = -Y_{11}$)

Giant dipole resonance (observed for A>16) $E \approx 77 \cdot A^{-1/3}\ \mathrm{MeV}$

λ=2 **vibration: quadrupole**

$$R(t) = R_0 + \sum_{\mu=-2}^{2} \alpha_{1\mu}(t) \cdot Y_{2\mu}(\theta,\varphi) = R_0 + \alpha_{22}Y_{22} + \alpha_{21}Y_{21} + \alpha_{20}Y_{20} + \alpha_{21}Y_{21} + \alpha_{2,\,-2}Y_{2,\,-2}$$

$$= R_0 + \alpha_{20}(t) \cdot Y_{20} = R_0 + \alpha_{20}(t)\ \frac{1}{4}\left(\frac{5}{\pi}\right)^{1/2} \left(3\cos^2\theta - 1\right)$$



because $\alpha_{2\mu} = 0$ for $\mu \neq 0$ (using appropriate coord. sys.)
(for ellipsoidal shape, *R* is a function of only θ)

The shape of the surface can be described by $Y_2^{\mu}(\theta,\varphi)$
$\mu = \pm2, \pm1, 0$. **In the case of an ellipsoid *R = R(θ)* hence $\mu = 0$.**

Quantization of quadrupole vibration is called a quadrupole phonon, $J^{\pi} = 2^{+}$. This mode is dominant.
For most even-even nuclei, a low lying state with $J^{\pi} = 2^{+}$ exists and near closed shells second harmonic states can be seen $J^{\pi} = 0^{+}, 2^{+}, 4^{+}$.

Giant quadrupole resonance (observed for A>16) $E \approx 63 \cdot A^{-1/3}\ \mathrm{MeV}$

λ=2 **vibration: quadrupole**

For a harmonic oscillation:

$$H = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 r^2 = \frac{1}{2}B\sum_{\mu}\left|\frac{d}{dt}\alpha_{2\mu}\right|^2 + \frac{1}{2}C\sum_{\mu}\left|\alpha_{2\mu}\right|^2$$

$$E_N = \left(N + \frac{5}{2}\right)\hbar\omega; \qquad \hbar\omega = \left(\frac{C}{B}\right)^{1/2}$$

Orthogonal transformation to get a diagonal form: $\sum_{\mu=-2}^{2} \alpha_{2,\mu} Y_2^{\mu}(\theta,\varphi) = \sum_{\mu=-2}^{2} a_{2,\mu} Y(\theta',\varphi')$

$$a_{2,1} = a_{2,-1} = 0 \qquad a_{2,2} = a_{2,-2}$$

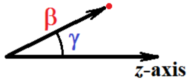
$$\sum_{\mu=-2}^{2}\left|\alpha_{2,\mu}\right|^2 = \sum_{\mu=-2}^{2}\left|a_{2,\mu}\right|^2 = \beta^2$$

Commonly new variables are introduced: $a_{2,0} = \beta\cos\gamma$**,** $a_{2,2} = \frac{1}{\sqrt{2}}\beta\sin\gamma$

The three main axis (*k* = 1,2,3) of the ellipsoid are then:

$$R_k - R_0 = \sqrt{\frac{5}{4\pi}} R_0 \beta \cos\left(\gamma - \frac{2\pi}{3}k\right)$$

These two variables can be considered as polar coordinates, thus every shape can be represented by a point in a 2D surface



There are β-vibrations, γ-vibrations one phonon, two phonon, etc…

The case of the two-phonon state

For every single-phonon state $J_1 = J_2 = 2^{+}$

Because of $|J_1 - J_2| \le J \le J_1 + J_2$ **we would expect $J = 0^{+}, 1^{+}, 2^{+}, 3^{+}, 4^{+}$**

But consider the „m" quantum numbers: $M = m_1 + m_2$

$J_1 = 2$	$J_2 = 2$		
m_1	m_2	M	
2	2	4	$J = 4$
2	1	3	
2	0	2	
2	-1	1	
2	-2	0	
1	1	2	$J = 2$
1	0	1	
1	-1	0	
0	0	0	$J = 0$

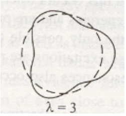
Only $J = 0^+, 2^+, 4^+$ occurs!

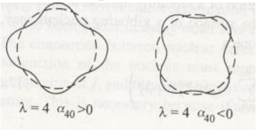
Note: Only non-negative m_1 and M values are shown. The table is symmetric for $M < 0$

Example: vibrational states in ^{114}Cd

Number of phonons			E	
$N = 2$	$2\hbar\omega$	4^+	1.283	two-phonon states
		2^+	1.208	
		0^+	1.132	
$N = 1$	$1\hbar\omega$	2^+	0.558	single-phonon state
$N = 0$	$0\hbar\omega$	0^+	0	ground state

$\lambda \geq 3$ **vibrations: octupole, hexadecupole etc.**





Octupole ($\lambda = 3$) modes with $J^\pi = 3$ can be observed in many nuclei.

The collective Hamiltonian (for $\lambda = 2$)

Rotation
(the main axis rotate)

Vibrational potential energy

$$\underbrace{H = \frac{1}{2}B\left(\dot{\beta}^2 + \beta^2\dot{\gamma}^2\right) + \frac{1}{2}\sum_{k=1}^{3}\theta_k\omega_k^2}_{\text{the total kinetic energy}} + \frac{1}{2}C\beta^2$$

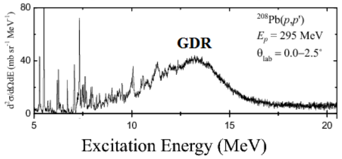
The energy: $E_{coll} = E_{vibr} + E_{rot}$

$$\boxed{E_{coll} = \sum_{\lambda}\left(n_\lambda + \frac{1}{2}\right)\hbar\omega_\lambda + \frac{\hbar^2}{2\theta}I(I+1)}$$

Experimental evidences for giant resonances

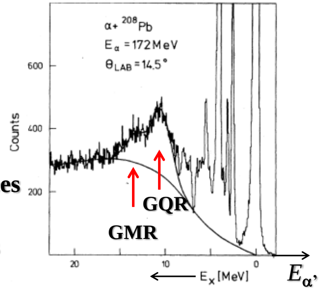
Mainly (inelastic) scattering experiments

Giant dipole resonance (p,p'), (e,e'), (γ,p) etc.



https://inspirehep.net/record/1242152/files/DCS_TST.png

(α,α'), (d,d') etc.



Morsch, Sükösd et al. Phys Rev. C 22 (1980) 489.

Giant Quadrupole (GQR) and Giant Monopole (GMR) resonances

Many different vibrational modes have been established also experimentally.