Nuclear Physics
(7^{th} lecture)

Content

- Gamma decay (contd.)
- Sum rules
- Measuring methods of the gamma decay constant
- Gamma-gamma angular correlation (multipolarity meas.)
- Nuclear models #1: liquid drop model
- Nuclear models #2: The Fermi-gas model

Gamma-decay

Summary of previous lecture:

1) Multipole expansion of electromagnetic waves

$\mathbf{B}_{l,m}^{(E)}(\mathbf{r}) = b_{l,m} f_l(\mathbf{kr}) \mathbf{L} Y_l^m(\vartheta,\varphi)$ **and** $\mathbf{E}_{l,m}^{(E)}(\mathbf{r}) = \frac{i}{k}\left(\nabla \times \mathbf{B}_{l,m}^{(E)}\right)$ **„electric”**

$\mathbf{E}_{l,m}^{(M)}(\mathbf{r}) = a_{l,m} f_l(\mathbf{kr}) \mathbf{L} Y_l^m(\vartheta,\varphi)$ **and** $\mathbf{B}_{l,m}^{(M)}(\mathbf{r}) = -\frac{i}{k}\left(\nabla \times \mathbf{E}_{l,m}^{(M)}\right)$ **„magnetic”**

These waves have good angular momentum and parity!

Electric transitions parity change: $\Delta\pi = (-1)^l$

Magnetic transitions parity change: $\Delta\pi = (-1)^{l+1}$

2) Selection rules **Angular momentum:** $|I_i - I_f| \le l_\gamma \le I_i + I_f$

Parity: $\pi_\gamma = \pi_1 \cdot \pi_2$

Note: since the „intrinsic” angular momentum of a photon is 1,
$l_\gamma \ge 1$ $\Longrightarrow$ **0 → 0 transitions are strictly forbidden.**

The gamma-decay (contd.)

Decay constant (transition probability)

Complicated quantum-electrodynamic calculation.
Approximation: only one unit charge changes its state.
Result: interaction operator

$$\lambda_E(l,m) = \frac{8\pi(l+1)}{\hbar l[(2l+1)!!]^2}\left(\frac{\omega}{c}\right)^{2l+1}\left|\langle\psi_f|M_{l,m}^E|\psi_i\rangle\right|^2$$ **for electric transitions**

$E_\gamma = \hbar\omega$ **final and initial wave functions**

Similar for magnetic transitions

Further approximations:

- **The** $\langle\psi_f|$ **and** $|\psi_i\rangle$ **wave functions contain** $Y_l^m(\vartheta,\varphi)$ **spherical harmonics, these can be integrated with the** $M_{l,m}^E$ **operator** → $S(I_i, I_f, l)$ **„statistical” factor**
- **The** m **quantum numbers averaged (no direct observation)**
- **The radial part of the wave functions = constant (!)**

The gamma-decay (contd.)

Finally we get the Weisskopf units:

$$\lambda_E(l) = 4{,}4 \cdot 10^{21}\frac{l+1}{l[(2l+1)!!]^2}\left(\frac{E_\gamma}{197}\right)^{2l}\left(\frac{3}{l+3}\right)\cdot R^{2l}\cdot S(I_i, I_f, l)$$

$$\lambda_M(l) = 1{,}9 \cdot 10^{21}\frac{l+1}{l[(2l+1)!!]^2}\left(\frac{E_\gamma}{197}\right)^{2l+1}\left(\frac{3}{l+3}\right)\cdot R^{2l-2}\cdot S(I_i, I_f, l)$$

Since this is dependent on units, R **should be in fm,** E_γ **in MeV.**
The results are in 1/s.

How do the transition probability change with multipolarity?

$$(E_\gamma R)^{2l} = (hc)^{2l}\left(\frac{R}{\lambda}\right)^{2l} \propto \left(\frac{R}{\lambda}\right)^{2l}$$

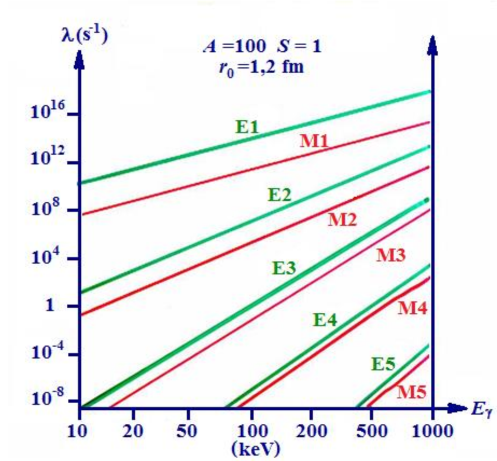
Take A **= 125 nucleus, and** E_γ **= 0,5 MeV**
$R = 1{,}2\cdot\sqrt[3]{125} = 6\ \text{fm}$ $\lambda = 400\ \text{fm}$ $\left(\frac{R}{\lambda}\right)^2 = 2{,}25\cdot 10^{-4}$

In atomic physics $R \approx 10^4\ \text{fm}$ **and** $\lambda \approx 8\cdot 10^8\ \text{fm}$ $\longrightarrow \left(\frac{R}{\lambda}\right)^2 \approx 10^{-10}$

Only E1 transitions occur for the atoms! Role of collisions!

The gamma-decay (contd.)

The trend of the Weisskopf units



Gamma-decay (contd.)

4) More complicated transitions

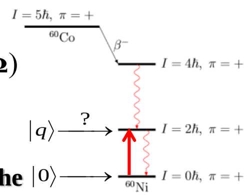
$$B(El, I_1 \to I_2) = \frac{1}{2I_1+1} \sum_{m_1,m_2,m} \left| \sum_p M^E_{l,m}(p) \right|^2$$

Reduced transition probability — Initial and final state „directions" — Single particle matrix elements

The $B(El)$ values are usually given in Weisskopf units (W.u.).
If $B(EL) \sim 1$ W.u. → single particle excitations
If $B(El) >> 1$ W.u. → collective excitation (involving many particles)

Sum rules

Excitation of the nucleus



The quadrupole operator: $\hat{\mathbf{Q}} = \hat{r}^2 \cdot \hat{Y}_2(\mathbf{\Omega})$

The possible quadrupole excitations: $|q\rangle = \hat{\mathbf{Q}}|0\rangle$

Usually $| q >$ is not an eigenfunction of the Hamiltonian, and even not normalized!

However, it can be expanded!
If $| \phi >$ is a complete normalized set of eigenfunctions then

$|q\rangle = \sum_f c_f |\phi_f\rangle$ Multiply this by $< \phi_i |$

$$\langle \phi_i | q \rangle = \sum_f c_f \langle \phi_i | \phi_f \rangle = \sum_f c_f \delta_{i,f} = c_i$$

$| q >$ can be normalized if $\sum_i |c_i|^2 = \sum_i |\langle \phi_i | q \rangle|^2 = S < \infty$

Sum rules (contd.)

Finally we have: $\sum_i \left|\langle \phi_i | \hat{\mathbf{Q}} | 0 \rangle\right|^2 = S$

This is the Thomas-Kuhn sum rule.
Here $\hat{\mathrm{Q}}$ can represent any multipole operator (not only quadrupole)

It can be calculated theoretically using simple assumptions!

For example for the dipole operator:

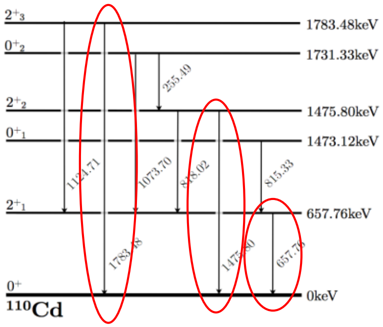
$$\sum_f \left|\langle \phi_f | D | 0 \rangle\right|^2 \propto \int \sigma_D(E) dE = \frac{2\pi e^2 \hbar^2}{mc} \cdot \frac{NZ}{A} \approx 60 \frac{NZ}{A} \quad \text{[MeV·mb]}$$

What does the sum rule mean physically?

$\left|\langle \phi_i | q \rangle\right|^2 = \left|\langle \phi_i | \hat{\mathbf{Q}} | 0 \rangle\right|^2 \sim$ how strongly the $< \phi_i |$ nuclear state can be excited from the ground state with the $\hat{\mathrm{Q}}$ operator

Sum rules (contd.)

There can be many excitations with the same multipolarity!



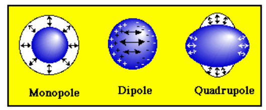
The sum rule describes the total possible strength of excitation (with any energy) for a multipole operator

$$\frac{\left|\langle \phi_i | \hat{\mathbf{Q}} | 0 \rangle\right|^2}{S} \leq 1$$ describes the contribution of one particular excitation to the sum rule (e.g. in %)

Sum rules (contd.)

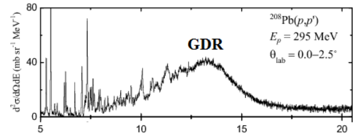
If only one $< \phi_i |$ contributed to S, then this particular state would „exhaust" 100% of the sum rule.

Giant multipole vibrations



„breathing" mode

Giant dipole resonance



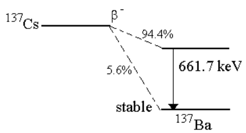
https://inspirehep.net/record/1242152/files/DCS_TST.png

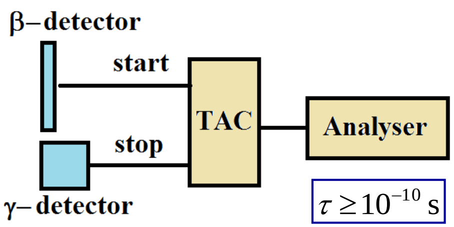
Measuring the gamma-decay constant

The exponential decay law $A(t) = A_0 \cdot e^{-\lambda t} = A_0 \cdot e^{-\frac{t}{\tau}}$

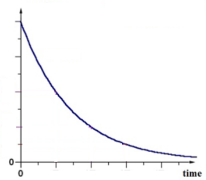
1) Direct measurement of life time

The method of delayed coincidences





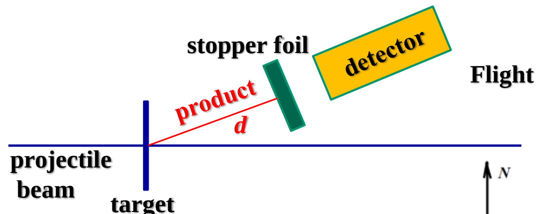
$\tau \geq 10^{-10}$ s



TAC = time to amplitude converter
(also time to digital converter TDC could be used)

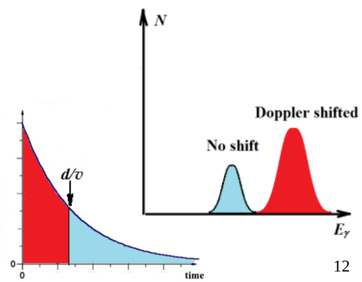
Measuring the gamma-decay constant

2) Using the Doppler shift $10^{-12} \leq \tau \leq 10^{-9}$ s



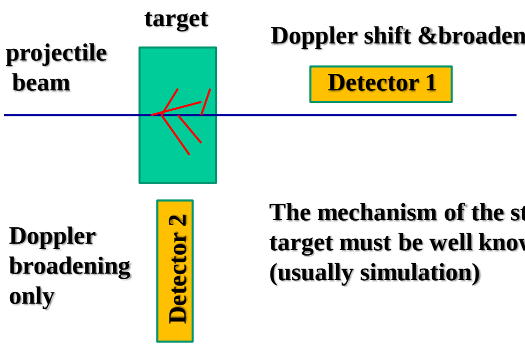
Flight time: $t = \frac{d}{v}$

During the flight: Doppler shift
After the stop: No Doppler shift



Measuring the gamma-decay constant

3) Doppler shift in the target $10^{-13} \le \tau \le 5\cdot 10^{-12}$ s

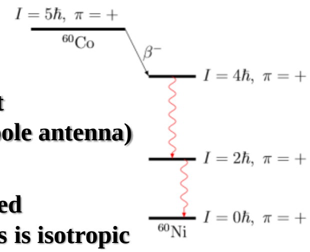


The mechanism of the stopping in the target must be well known (usually simulation)

4) From line-width $\Delta E_\gamma \cdot \tau \approx \hbar$ **(if ΔE_γ > detector resolution)**

Gamma-gamma angular correlation

A method for determining the multipolarity



The idea:
The angular distribution of different multipolarities are different (e.g. dipole antenna)

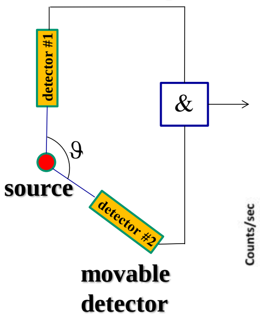
The problem:
The nuclei in a sample are not aligned
→ observation of single gamma-rays is isotropic

The solution:
Detect two consecutive gamma-rays! The detection of the first gamma-ray fixes a direction → observation of the second gamma-ray will not be isotropic respective to the direction of the first one!

The angle between the two γ-rays depends on their multipolarity!

Gamma-gamma angular correlation (contd.)

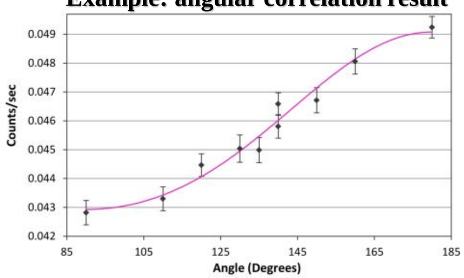
Coincidence experiment



$$W(\vartheta) = \sum_i a_i \cdot \cos(2i\cdot\vartheta)$$

The number and the values of the a_i coefficients are characteristic for the multipolarities involved

Example: angular correlation result



https://wiki.umn.edu/pub/MXP/AngularCorrelationofGammaRaysfromCobalt60/graph.jpg

Nuclear models

The problem:

Unlike the atom, the nucleus cannot be described „exactly”, because…

a) … the nucleus is held together by the two-body interactions between its constituents; there is no central source for the nuclear potential;

b) …we don't know exactly the nucleon-nucleon interaction.

The solution:

We use simplified models. The pay-off is that these models describe the behaviour only partly.

1) The liquid drop model

We have treated it already. That led to the Weizsäcker-formula. It describes well the binding energy. But only that!

Assumptions:

- **Nuclear forces are attractive and short-ranged**
  → the density is constant (like in a liquid)
- **The nucleus is sphere-shaped (because of the surface tension)**
  → the surface energy is proportional to the surface
- **The nucleus is homogeneously charged**
  → Coulomb term
- **The constituents are fermions (Pauli principle)**
  → asymmetry energy term
- **Pairing energy can be empirically taken into account**

$$B = b_V A - b_F \cdot A^{2/3} - b_C \cdot \frac{Z^2}{A^{1/3}} - b_A \cdot \frac{(N-Z)^2}{A} + b_P \cdot \delta \cdot A^{-1/2}$$

17

2) The Fermi-gas model

Since the nucleons are fermions, we can try to set up a model of fermion „gas".

Assumption: the fermions are „closed" in a spherical potential well, but they move „freely" inside .

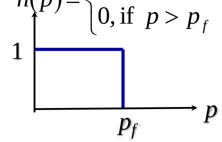
What can be expected from this model?

- **Describe only the kinetic energy contribution of the nucleons**
- **Since quantum mechanics is used, the „asymmetry" term might be described**
- **Some surface effects can also be expected**

The ground state:

- **Since they are fermions, only one particle in a state (Pauli exclusion principle)**
- **Since ground state → the lowest energy states are filled**
- $p = \hbar k$

$$n(p) = \begin{cases} 1, \text{ if } p \le p_f \\ 0, \text{ if } p > p_f \end{cases}$$



18

The number of states in the phase-space:

$$\mathrm{d}\mathcal{N} = 2 \cdot \frac{\mathrm{d}^3\mathbf{r} \cdot \mathrm{d}^3\mathbf{p}}{(2\pi\hbar)^3} = 2 \cdot \mathrm{d}^3\mathbf{r} \cdot \frac{p^2 \cdot \mathrm{d}p}{(2\pi\hbar)^3} \mathrm{d}\Omega_p$$

↑ Because of the spin

$$\mathcal{N} = \int n(p) \mathrm{d}\mathcal{N} = 2 \iint n(p) \cdot \mathrm{d}^3\mathbf{r} \cdot \mathrm{d}^3\mathbf{p} = 2 \frac{V}{(2\pi\hbar)^3} 4\pi \int_0^{p_f} p^2 \cdot \mathrm{d}p = 2 \frac{V}{(2\pi\hbar)^3} \cdot \frac{4\pi p_f^3}{3}$$

For the number of protons and of neutrons we have

$$Z = 2 \frac{V}{(2\pi\hbar)^3} \cdot \frac{4\pi p_{fp}^3}{3} \qquad N = 2 \frac{V}{(2\pi\hbar)^3} \cdot \frac{4\pi p_{fn}^3}{3}$$

Using $R = r_0 \sqrt[3]{A}$ **and** $V = \frac{4\pi}{3} R^3 = \left(\frac{4\pi}{3} r_0^3\right) \cdot A$ **we get :**

$$p_{fp}^3 = \frac{Z}{A} \cdot \frac{(2\pi\hbar)^3}{2 \cdot \frac{4\pi}{3} r_0^3} = 3\pi^2 \left(\frac{\hbar}{r_0}\right)^3 \cdot \frac{Z}{A}, \quad \text{and} \quad p_{fn}^3 = 3\pi^2 \left(\frac{\hbar}{r_0}\right)^3 \cdot \frac{N}{A}$$

19

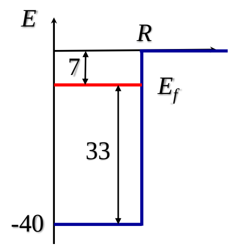
To get some numerical estimation we approximate: $\frac{Z}{A} \approx \frac{N}{A} \approx \frac{1}{2}$

we get $p_{fp} \approx p_{fn} = \left(\frac{3\pi^2}{2}\right)^{1/3} \cdot \frac{\hbar}{r_0}$ **Contains only known constants!**

$r_0 \approx 1{,}2\ \mathrm{fm}$

The highest kinetic energy at the Fermi-level: $E_f = \frac{p_f^2}{2M} \approx 33\ \mathrm{MeV}$

Since the average binding energy of a nucleon is ~ 7 MeV, we get the following picture for the nuclear potential:



20