Nuclear Physics
(6^{th} lecture)

Content

- **Alpha decay, energy considerations**
- **Half-life of the alpha-decay, transmission coefficient**
- **Alpha spectroscopy factor, and the alpha spectrum**
- **The gamma-decay**
- **Multipole expansion of electromagnetic waves**

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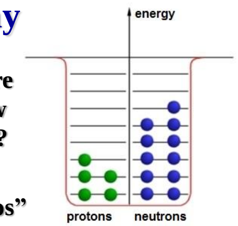
The alpha-decay

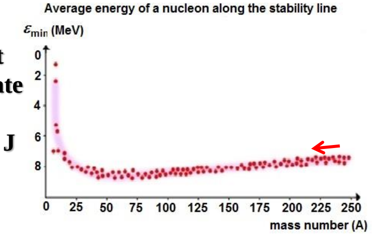
Energy considerations

Problem: the nucleons are bound, therefore no single nucleon emission is possible. How can 4 nucleons (alpha-particle) be emitted?



Several factors make this possible:

- **The nucleus does not „step", but „jumps" down in the nuclear valley**
- **The emitted 4 nucleons get out in a strongly bound state (more bound than in the parent) B_{alpha} = 4,533·10^{-12} J**



Where can alpha-decay be expected?

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The alpha-decay (contd.)

$${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

The Q reaction energy: $\frac{Q}{c^2} = M(Z,A) - M(Z-2,A-4) - M(2,4)$

Proton & neutron number: conserved → diff. only in binding

$Q = B(Z-2,A-4) + B(2,4) - B(Z,A)$ **(B is the binding energy)**

Energy should be available for the decay: $Q > 0$

$$B(Z-2,A-4) + 4{,}533 \cdot 10^{-12}\,[\mathrm{J}] - B(Z,A) > 0$$

Substituting in the Weizsäcker formula (in 10^{-12} J units) we get:

$$-2{,}85\left((A-4)^{\frac{2}{3}} - A^{\frac{2}{3}}\right) - 0{,}11\left(\frac{(Z-2)^2}{(A-4)^{\frac{1}{3}}} - \frac{Z^2}{A^{\frac{1}{3}}}\right) - 15{,}16\frac{(A-2Z)^2}{A(A-4)} - 5{,}547 \geq 0$$

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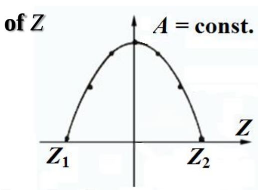
$$-2{,}85\left((A-4)^{\frac{2}{3}} - A^{\frac{2}{3}}\right) - 0{,}11\left(\frac{(Z-2)^2}{(A-4)^{\frac{1}{3}}} - \frac{Z^2}{A^{\frac{1}{3}}}\right) - 15{,}16\frac{(A-2Z)^2}{A(A-4)} - 5{,}547 \geq 0$$

For A = const. this is a parable in function of Z
The coefficient of Z^2 is

$$a = -\frac{60{,}64}{A(A-4)} - 0{,}11\left(\frac{1}{(A-4)^{\frac{1}{3}}} - \frac{1}{A^{\frac{1}{3}}}\right) < 0$$



for any A>4, → the inequality leads to: $Z_2 \leq Z \leq Z_1$

Real roots exist only if the discriminant is positive!
This occurs only for A >90 !

Indeed, alpha-decay occurs only for heavy nuclei!

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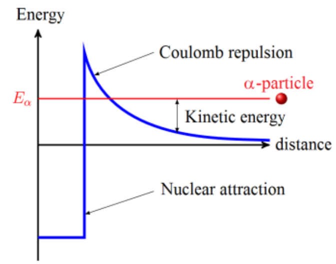
Half life of alpha-decaying nuclei

Problem: if energetically possible, why dont they decay instantly?

Imagine the inverse process: an α-particle approaches the nucleus.

The Coulomb-barrier is a barrier also „from inside"!

The decay can occur only via quantum-mechanical tunnel effect!



Two factors play a role: $\lambda = \lambda_0 \cdot T$

Decay constant (→ λ)

α-cluster formation probability (→ λ_0)

Transmission probability (→ T)

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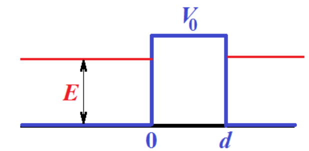
Transmission probability: $T = \frac{j_{through}}{j_{in}}$ ← **current densities**

$$\mathbf{j} = \frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) \propto |\psi|^2 \mathbf{v}$$ **Using** $\mathbf{p} = m\mathbf{v} = \hbar \mathbf{k}$

$$T = \frac{|\psi_{through}|^2 \cdot k_{through}}{|\psi_{in}|^2 \cdot k_{in}}$$

One dimensional square potential

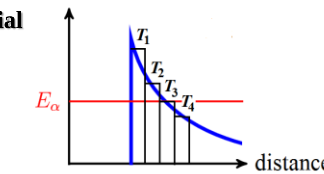
$$T \propto \exp\left(-\frac{2}{\hbar}\sqrt{2m(V_0 - E)} \cdot d\right)$$



One dimensional arbitrary potential

$$T = T_1 \cdot T_2 \cdot T_3 \cdot \ldots$$

$$T \propto \exp\left(-\frac{2}{\hbar}\int \sqrt{2m(V(r) - E)}\,dr\right)$$



Three dimensional central potential (no angular dependence)

$$\psi(r, \vartheta, \varphi) = \frac{R(r)}{r} \cdot Y_l^m(\vartheta, \varphi)$$

BUT! In addition to the *V(r)* central potential also the „centrifugal potential" contributes

$$V_l = \frac{l(l+1)\hbar^2}{2mr^2}$$

Therefore inside the potential hill we have

$$k^2 = \frac{2m}{\hbar^2}\left(E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2}\right)$$ **Here** $m = \frac{m_1 m_2}{m_1 + m_2}$ **the reduced mass**

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$$T = \frac{|\psi_{through}|^2 \cdot k_{through}}{|\psi_{in}|^2 \cdot k_{in}} = \frac{k_{through}\int \frac{R^2_{through}(r)}{r^2} r^2 dr}{k_{in}\int \frac{R^2_{in}(r)}{r^2} r^2 dr} \cdot \frac{\int \left|Y^m_{l,through}(\vartheta,\varphi)\right|^2 d\Omega}{\int \left|Y^m_{l,in}(\vartheta,\varphi)\right|^2 d\Omega}$$

Finally ($\uparrow$ **= 1**)

$$T = \frac{k_{through}\int R^2_{through}(r)dr}{k_{in}\int R^2_{in}(r)dr}$$ **Deduced back to one-dimensional case!**

Coulomb potential: $V(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{r}$

$T \propto e^{-G}$ **where** $G = \frac{2\sqrt{2m}}{\hbar}\int_R^{R'}\left(\sqrt{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{r} - E}\right)dr$

Gamow-factor

$$G = \frac{2}{\hbar}\sqrt{\frac{2m}{E}}\left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0}\right) \cdot \gamma\left(\frac{E}{V_C}\right)$$ **where** $\gamma\left(\frac{E}{V_C}\right) = \arccos\sqrt{\frac{E}{V_C}} - \sqrt{\frac{E}{V_C}\left(1 - \frac{E}{V_C}\right)}$

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The half life $T_{1/2} = \frac{\ln 2}{\lambda} \propto \frac{1}{T_\alpha} \propto e^G$

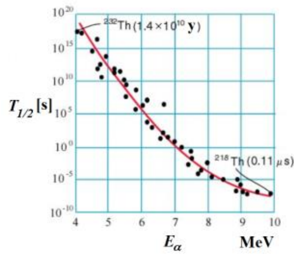
A few data

	$T_{1/2}$	E_a (MeV)	T_a
^{212}Po	0,3 μs	8,78	$1{,}32\cdot10^{-13}$
^{224}Ra	3,6 day	5,7	$5{,}9\cdot10^{-26}$
^{144}Nd	$2\cdot10^{15}$ year	1,83	$2{,}18\cdot10^{-42}$

Empirically
H. Geiger and G. M. Nutall (1911)

$$\log T_{1/2} = a + b\cdot\log E_\alpha$$

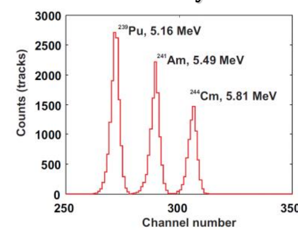
Important for half-life estimation !

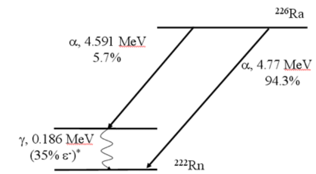


The alpha spectroscopy factor (λ_0) $\lambda = \lambda_0 \cdot T$

Describes the probability of an alpha-cluster formation inside the nucleus!
Can be studied via alpha-transfer reactions e.g. (d,^{6}Li)

The alpha spectrum: **contains lines with well defined energies**
Suitable for analysis!





The gamma-decay

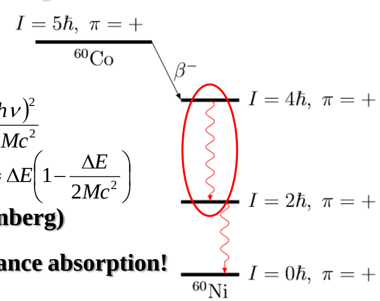
Gamma radiation → electromagnetic (E.M.) radiation
Electric charge density changes in time → „induced" E.M. waves
Conservation laws:
Composition (Z,A) does not change
Momentum conservation:
The nucleus recoils $p_\gamma = \frac{h\nu}{c}$
Energy conservation:

$$\Delta E = E_i - E_f = h\nu + \frac{p^2}{2M} = h\nu + \frac{(h\nu)^2}{2Mc^2}$$

Since $\Delta E << Mc^2$ **we get** $h\nu \approx \Delta E\left(1 - \frac{\Delta E}{2Mc^2}\right)$

Line width: $\Gamma\cdot\tau \geq \frac{\hbar}{2}$ **(Heisenberg)**

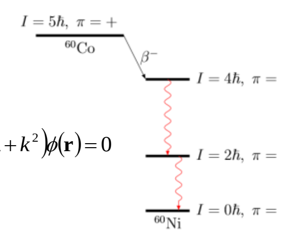
Usually $\Gamma << \frac{\Delta E}{2Mc^2}$ **no resonance absorption!**



Exception: Mössbauer effect
(the whole crystal recoils → M is very large

The gamma-decay (contd.)

Conservation laws:
Angular momentum?
Parity?



Maxwell's Equations
Free radiation → wave equation $(\Delta + k^2)\phi(\mathbf{r}) = 0$
„Multipole composition" needed

Solution: $\phi(\mathbf{r}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \alpha_{l,m} f_l(\mathbf{kr}) Y_l^m(\vartheta,\varphi)$

$\phi_1(\mathbf{r}) = \mathbf{r}\cdot\mathbf{E}(\mathbf{r}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{l,m} f_l(\mathbf{kr}) Y_l^m(\vartheta,\varphi)$ **and** $\mathbf{B}(\mathbf{r}) = -\frac{i}{k}(\nabla\times\mathbf{E})$

$\phi_2(\mathbf{r}) = \mathbf{r}\cdot\mathbf{B}(\mathbf{r}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} b_{l,m} f_l(\mathbf{kr}) Y_l^m(\vartheta,\varphi)$ **and** $\mathbf{E}(\mathbf{r}) = \frac{i}{k}(\nabla\times\mathbf{B})$

The solutions fall into two categories:
$\mathbf{r}\cdot\mathbf{E}(\mathbf{r}) = 0$ **called „electrical" transitions (transversal electric)**
$\mathbf{r}\cdot\mathbf{B}(\mathbf{r}) = 0$ **called „magnetic" transitions (transversal magnetic)**

The solutions:

$\mathbf{B}_{l,m}^{(E)}(\mathbf{r}) = b_{l,m} f_l(\mathbf{kr})\mathbf{L}Y_l^m(\vartheta,\varphi)$ **and** $\mathbf{E}_{l,m}^{(E)}(\mathbf{r}) = \frac{i}{k}\left(\nabla\times\mathbf{B}_{l,m}^{(E)}\right)$ **„electric"**

$\mathbf{E}_{l,m}^{(M)}(\mathbf{r}) = a_{l,m} f_l(\mathbf{kr})\mathbf{L}Y_l^m(\vartheta,\varphi)$ **and** $\mathbf{B}_{l,m}^{(M)}(\mathbf{r}) = -\frac{i}{k}\left(\nabla\times\mathbf{E}_{l,m}^{(M)}\right)$ **„magnetic"**

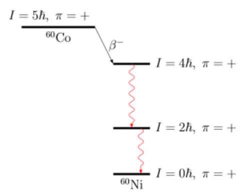
Here $\mathbf{L} = \mathbf{r}\times\mathbf{p} = \hbar(\mathbf{r}\times\mathbf{k})$

These waves have good angular momentum and parity!

Electric transitions parity change: $\Delta\pi = (-1)^l$

Magnetic transitions parity change: $\Delta\pi = (-1)^{l+1}$

Selection rules



Angular momentum $|I_i - I_f| \le l_\gamma \le I_i + I_f$

and $m_\gamma = m_i - m_f$

Parity: $\pi_\gamma = \pi_1 \cdot \pi_2$