Nuclear Physics
(2^{nd} lecture)

Content

- Discovery of the nucleus, Rutherford's experiment
- Size of the nucleus, Hofstädter experiments.
- The composition of the nucleus.
- Angular momentum and parity
- Electromagnetic moments
- NMR and MRI
- Excited states, level schemes

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Discovery of the nucleus

XIX century:
- Conservation of energy (thermodynamics, industrial revolution)
- Chemical elements do not transform (Mendeleev system)
- Avogadro's discovery, the concept of mole

  Number of particles in a molar mass: $N_A = 6 \cdot 10^{23}$

  Atoms are only „work-models", not reality (yet)

  Consequence: size of „model-atoms" can be calculated!

Example: size of gold atoms

Molar mass of Au: 197, i.e. 197 g gold contains $6 \cdot 10^{23}$ atoms

Density of Au : 19,3 g/cm^3, i.e. the volume of 197 g gold is:

$$(197/19{,}3) \sim 10 \text{ cm}^3$$

Therefore the volume of one single atom:

$$(10 \text{ cm}^3)/(6 \cdot 10^{23}) \sim 16 \cdot 10^{-24} \text{ cm}^3$$

The size of a „cube" containing one atom: $\sqrt[3]{16 \cdot 10^{-24}} = 2{,}52 \cdot 10^{-8}$ cm

The radius of the atoms are ~ 10^{-10} - 10^{-9} m.

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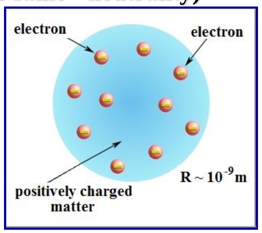
Discovery of the nucleus (contd.)

Atoms are electrically neutral, but positively and negatively charged particles can originate from them

1897: discovery of the electron (J. J. Thomson, q/m measurement)
- it is inside every atom,
- small mass,
- negatively charged

Consequence: the positive charge must have larger mass.
Mathematically: for the electron $|q|/m_{electron} >> |q|/M_{positive}$
(since the charges must be the same - neutrality)

Thomson's model of the atom
(pudding model)



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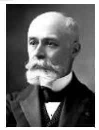
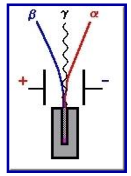
Discovery of the nucleus (contd.)

1896: discovery of the natural radioactivity (H. Becquerel)

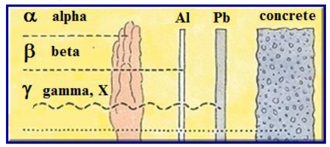


Around the beginning of the 20^{th} century:
- high energy particles are emitted (α, β, γ)
- elements will be transformed into each-other



the electrical charge of the particles are different
- α – particles: heavy, He^{2+} ions
- β – particles: high energy electrons
- γ - radiation: electromagnetic (photons)

The penetration range of the radiations are very different!



Surprising, since the matter is electrically neutral!

E. Rutherford: why the α–particles cannot fly through even a thin paper sheet?

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Discovery of the nucleus, Rutherford's experiment

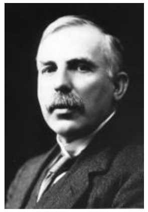
1911: Rutherford's experiment

- **Interaction of α– particles with thin gold (Au) foil**

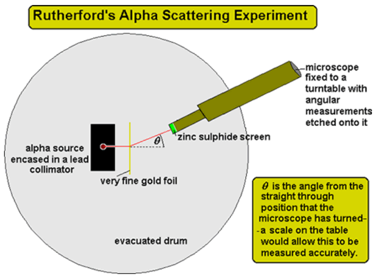
(performed by Geiger and Marsden)

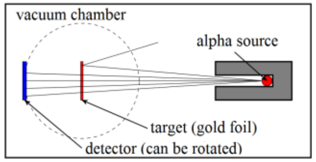
Why gold?

- **gold could be hammered very thin (few atomic layers)**



Sir Ernest Rutherford (1871-1937)
Nobel-Prize in Chemistry 1908





http://www.cyberphysics.co.uk/topics/atomic/Rutherford/rutherford.gif

What could be expected?

The positively charged, heavy part of the Thomson-atom repels the α- particles with E_{alpha} kinetic energy – Coulomb-potential

The maximal „height" of the potential hill:

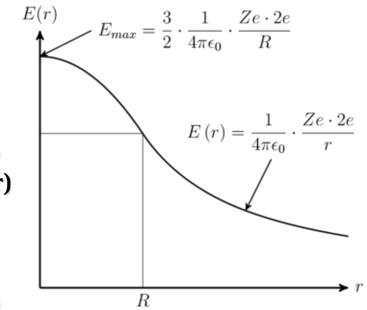
$$E_{\max} = \frac{3}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{R_{atom}}$$

Here Z_1= 2 (He atomic number)
Z_2= 79 (Au atomic number)

$$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \left[\frac{\mathrm{J \cdot m}}{\mathrm{Cb}^2}\right]$$

e = 1,6 ·10^{-19} Cb

R_{atom}~10^{-10} m (seen earlier)



With all these E_{max}~ 5,5 ·10^{-16} J

But: E_{alpha}~ 7700 ·10^{-16} J

Like cannonball through paper!

http://www.reak.bme.hu/fileadmin/user_upload/felhasznalok/sukosd/letoltesek/Rutherford_en/Rutherford.htm

Discovery of the nucleus, Rutherford's experiment

Observation:

There were few backscattered particles!

Conclusion:

The potential hill must be „higher", than the energy of the α-particle, i.e.:

$$E_{alpha} < \frac{3}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{R} \text{ from where: } \boxed{R < \frac{3}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{E_{alpha}}}$$

Substituting the data we get: R<10^{-14} m, i.e. 10000 times smaller than the radius of the atom!

In the atoms the mass and the positive charge is concentrated in the very small nucleus!

Rutherford's experiment, discussion

Rutherford's results give only an upper limit for the size of the nucleus, they do not give the actual size!

Further questions:

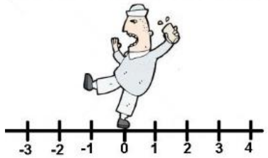
- **Is the large angle scattering (backscatter) caused by one single scattering event, or results from several small angle scattering?**
- **Is the cause really the Coulomb-interaction (or „hard sphere")?**
- **How does it depend on the atomic number of the target atom?**
- **How does it depend on the kinetic energy of the alpha-particle?**
- **etc…**

Rutherford and co-workers thoroughly studied these questions. Good example, how an experiment should be performed!

Rutherford’s experiment, discussion

a) Single or multiple scattering?

„Drunken sailor” model (random walk in one dimension)



If you are away from the pier’s edge by σ steps, after how many (N) tottering steps will you fall into the sea in average?

$$\boxed{\sigma = \sqrt{\frac{2}{\pi}} \cdot \sqrt{N}}$$

Suppose that the alpha-particle scatters on one atomic layer by ϑ angle in average. Then after N scattering the final average angle will be:

$$\boxed{\sigma_\vartheta = \vartheta \cdot \sqrt{\frac{2}{\pi}} \cdot \sqrt{N}}$$

Naturally $N \sim d$ (the number of atom layers is proportional to the thickness of the foil)

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Rutherford’s experiment, discussion

It follows, that the probability of the deviation with a given angle is

- **for single scattering** $\boxed{P(\vartheta) \sim d}$ **(linear)**
- **for multiple scattering** $\boxed{P(\vartheta) \sim \sqrt{d}}$ **(square root)**

Geiger and Marsden measurements:

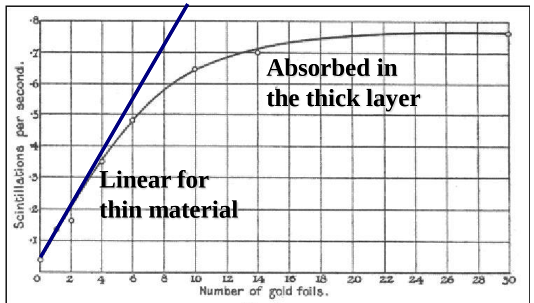

FIG. 2.

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Rutherford’s experiment, discussion

b) Is that really Coulomb-scattering?

Maybe some other type of interaction (e.g. „hard sphere”)?
How can this be determined?

Comparison of experimental result with theoretical models!
(still the main method of nuclear and particle physics!!)

Rutherford theoretically deduced the angular distribution of scattered particles by Coulomb field of a point like heavy target

Detection probability $\sim \left(\frac{Z_1 e \cdot Z_2 e}{4 \cdot E_\alpha}\right)^2 \cdot \frac{1}{\sin^4(\vartheta/2)}$ **Agreement with the experiments!**

Different interactions result in different angular distributions!

c) They also verified the dependence on the target atomic number (using different target foils)…

d) … and the dependence on the energy of the alpha particles (using different alpha-sources)

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Rutherford’s experiment, discussion

Mother Nature is benignant and sometimes helps the researchers! Now we know that Rutherford was lucky from two points of view!

1) **Rutherford could not be aware of the wave nature of the particles, since it was discovered much later by de Broglie. However, the de Broglie wavelength of the used alpha-particles is by chance comparable to the size of the nucleus!** $\boxed{\lambda = \frac{h}{p} \sim 10^{-14}\ \mathrm{m}}$ **So he was able to „see” the nucleus. ☺**
2) **For particles one should use quantum physics in the calculations, but Rutherford could only use the classical physics in 1911. The quantum physics was developed much later. BUT! The Coulomb interaction is the only one, where the classical and quantum-physical treatment gives exactly the same result for the angular distribution of scattering! ☺**

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The size of the nucleus, Hofstädter's experiment

Rutherford's results gave only an upper limit for the size of the nucleus, they did not give the actual size!
Can we measure also the size? Or see even smaller details?
Yes! Using high-energy electrons (very small wavelength) Hofstädter could determine even the charge-distribution inside the nucleus!

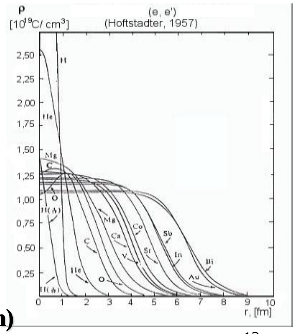
Results:

- **central charge density ~ constant**
- $\boxed{R = r_0 \cdot A^{1/3}}$**, where**
- $r_0 = 1{,}23 \cdot 10^{-15}$ **m = 1,23 fm and**
- ***A* is the number of nucleons.**

The form of density can be described

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{d}}}$$ **(Fermi-function)**

***R* is the radius of the nucleus, (~$r_0 \cdot A^{1/3}$)**
***d* is the surface „diffuseness" (d~2,4 fm)**



The Rutherford model of the atom

Rutherford's results showed that the nucleus had two distinct parameters: the mass (*M*) and the charge (*Q*).
The nucleus of the hydrogen had the smallest mass and charge
$m = 1{,}67262178 \cdot 10^{-27}$ **kg, and**
$q = 1{,}602 \cdot 10^{-19}$ **C = +*e* (positive elementary charge)**

The charge of the nuclei of the heavier elements: $Q = Z \cdot q$, where Z = 1, 2, 3, 4…. 92 (atomic number)

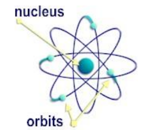
The mass of the nuclei of the heavier elements: $M \sim A \cdot m$, where A = 1, 2, 3, 4…. 238... (mass number)

However, $A > Z$ for all, except the hydrogen.

Therefore the first model of the nucleus was:
„*A*" protons to make the mass
„*A-Z*" inner electrons to compensate for the excess charges

To make the atom neutral we need *Z* „outer" electrons, that are orbiting around the massive nucleus, like planets around the Sun.

This was the Rutherford model of the atom



The Rutherford model of the atom (contd.)

Problems with the Rutherford model:

- **The „revolving" electrons should emit radiation (accelerating charge) $\longrightarrow$ no stable trajectories are possible**
- **Rutherford: where is the „small hydrogen atom"? (small, as a proton, but neutral…) James Chadwick (co-worker of Rutherford) tried to find it, with no success in the 20's**
- **The mass is not „exactly" $A \cdot m$, only approximately**

1927: According to the quantum mechanics (Heisenberg uncertainty principle), electrons cannot be confined in such a small place, because Δp becomes too large, and the Coulomb forces cannot keep them inside the nucleus $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

Chadwick abandoned the search for the small hydrogen atom, but the puzzle of the composition of the nucleus persisted.

1932: F. Joliot-Curie & Irene Curie (Paris): α+Be $\longrightarrow$ radiation
γ-radiation? Contradictions

1932 J. Chadwick: discovery of the neutron (Nobel-price 1935)

The composition of the nucleus

- ***Z* proton (atomic number)**
- ***N* neutron**
- ***A* = *N*+*Z* (mass number, nucleonic number)**

Nucleon: common name for protons and neutrons

	proton	neutron	spin
mass	$1{,}67265 \cdot 10^{-27}$ kg	$1{,}67495 \cdot 10^{-27}$ kg	½ $\hbar$
charge	+e	0	½ $\hbar$

Notation: $\boxed{{}^{197}_{79}\mathrm{Au}}$ $N = A - Z = 197 - 79 = 118$

Terms used:

- **nuclei with the same atomic number: ISOTOPES**
- **nuclei with the same mass number: ISOBARS**
- **nuclei with the same neutron number: ISOTONES (seldom)**

More parameter of the nucleus: angular momentum

Principle: every single nucleon is a fermion $\longrightarrow$ behaves like an electron in the atom:

It has spin (s) and orbital angular momentum (l) j = l + s (vectors)

Quantum mechanical addition of angular momenta: $j = l \pm ½$

Since l is integer, j is always half-integer.

Total angular momentum of the nucleus then: $\mathbf{J} = \sum_{i=1}^{A} \mathbf{j}_i$ (vectors)

Gets quite complicated!

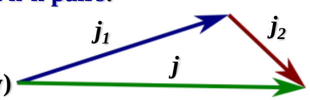
For even A: J is integer

For odd A: J is half-integer

HOWEVER! Fortunately, Nature makes things simpler!

Nucleons are paired: forms p-p and n-n pairs!

The resulting angular momentum of a pair could be $|j_1 - j_2| \le j \le j_1 + j_2$ (triangle inequality)



But the pairs choose always the $j = 0$!! (in ground state)

More parameter of the nucleus: angular momentum

Consequences:

- For every even-even nucleus $J = 0$ in ground state!
- For even-odd nucleus $J = j$
  (where j is the angular momentum of the unpaired nucleon)
- For odd-odd nucleus $\mathbf{J} = \mathbf{j}_1 + \mathbf{j}_2$ $|j_1 - j_2| \le J \le j_1 + j_2$
  (the vector sum of the two unpaired nucleons)

More parameter of the nucleus: parity

Remember: in quantum mechanics the parity is a good quantum number: $\Psi(-\mathbf{r}) = \pm\Psi(\mathbf{r})$ (symmetry to mirroring the axis)

parity ↗ (can be + or -)

More parameter of the nucleus: electric moments

Since parity is a good quantum number:

$\Psi(-\mathbf{r}) = \pm\Psi(\mathbf{r}) \longrightarrow |\Psi(-\mathbf{r})|^2 = |\Psi(\mathbf{r})|^2$

the density distribution is symmetric: $\rho(-\mathbf{r}) = \rho(\mathbf{r})$

Therefore all odd-ranked electric moments should be zero!

For example the dipole moment: $\mathbf{d} = \int \mathbf{r} \cdot \rho(\mathbf{r}) \cdot d^3 r = 0$

The first non-vanishing electric moment is the quadrupole moment: $Q = \int (3z^2 - r^2) \cdot \rho(\mathbf{r}) \cdot d^3\mathbf{r}$

More parameter of the nucleus: magnetic moment

Dipole magnetic moments can be originated from two sources:

- „revolving" charge (e.g. protons on $l > 0$ orbits)
- Nucleons have „intrinsic" magnetic moments:
  μ_p = 2,7928456 μ_N
  μ_n = –1,91304185 μ_N,
  where μ_N is the „nuclear magneton". $\mu_N = \frac{e \cdot \hbar}{2M_p} = 3{,}1525 \cdot 10^{-8} \frac{\text{eV}}{\text{T}}$

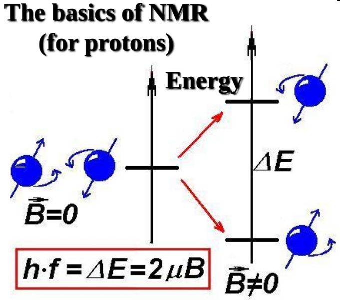
In general: $\boxed{\boldsymbol{\mu} = g \cdot \mathbf{J} \cdot \mu_N}$ g is called: „gyromagnetic factor"

Use of the nuclear magnetic moment: NMR, MRI

NMR: Nuclear Magnetic Resonance,

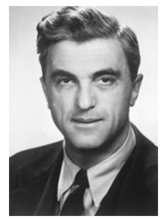
MRI: Medical Resonance Imaging

The basics of NMR (for protons)



(not only protons are suitable !)

1938-1945:
Felix Bloch and Edward Purcell developed the NMR method

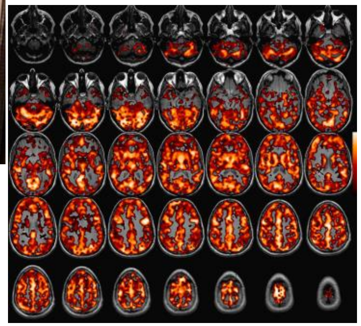


Felix Bloch
(the first DG of CERN)

Uses two important parameters of the nuclei:
- angular momentum ($\frac{1}{2}\hbar$ for protons)
- magnetic dipole moment

The MRI-scanner

For example: cranial tomography



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Excited states, nuclear level schemes

The nucleus is a quantum mechanical system:

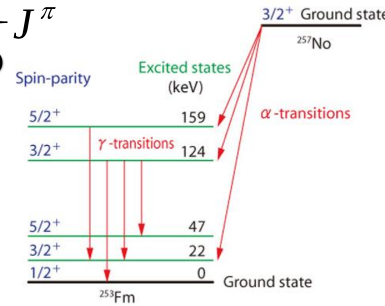
It has different states: ground state and excited states

Every state is characterized usually by the following

- **Excitation energy (above the ground state)**
- **Angular momentum**
- **Parity**
- **Decay modes (α,β,γ…)**
- **Half life**
- **etc…**

J^{π}

Nuclear level schemes



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