HUVINETT

Nuclear fission

<u>Course on Nuclear Fundamentals</u>
4th lecture

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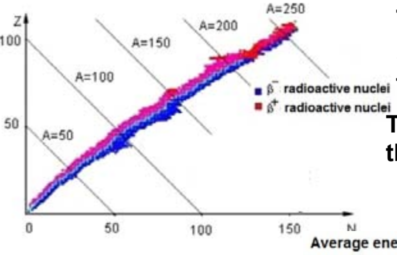
Reminder: The „Nuclear Energy Valley”

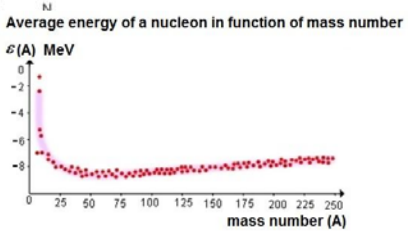


The place of Z_{min} on the (N,Z) „map” is between the blue and purple region

This helped to understand the radioactive decays!

ε_{min} in function of the (A) mass number

This helps to understand the energy production from the nuclei (nuclear energy production)



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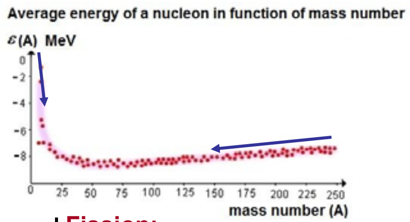
Options for releasing nuclear energy

<u>Fusion</u>
(make larger nuclei from smaller ones)

For example:
$^2H + ^3H \longrightarrow ^4He + n$
(Q = 17,6 MeV)

<u>Fission</u>
(make smaller nuclei from larger ones)



Fusion:
- big change in $\frac{\text{energy}}{\text{nucleon}}$ (~ 2 … 5 MeV/nucleon)
- few nucleon (~ 2 ... 5),

Total energy gain ~ 2–18 MeV

Fission:
- small change in $\frac{\text{energy}}{\text{nucleon}}$ (~0,85 MeV/nucleon)
- many nucleons (~236),

Total energy gain ~ 200 MeV

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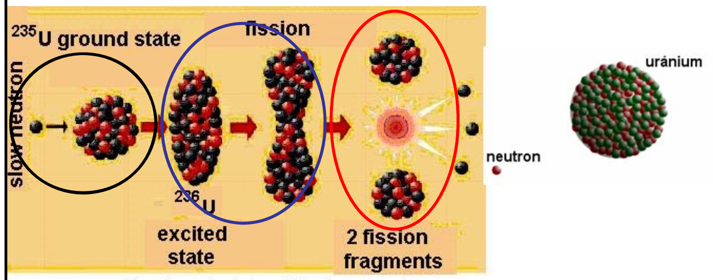
The fission process



Neutron induced fission

1) The nucleus gets excited (e.g. after neutron absorption)
2) It gets deformed, …
3) …then scissions to 2 parts, and emits a few neutrons

Some heavy nuclei undergo spontaneous fission, (step 1 is missing for them)

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The „equation" of the fission reaction:

$$^{235}\mathrm{U} + \mathrm{n} \longrightarrow \mathbf{Fragment_1} + \mathbf{Fragment_2} + 2{,}4\ \mathrm{n}$$

Two obvious questions:

1) Why don't we write down the results (the daughter nuclei) exactly?
2) What does $2{,}4\ \mathrm{n}$ signify? Number of neutrons can be only integer number!

The answers:

1) The composition (Z,A) of the fragments cannot be defined more precisely, since the fission process is stochastic also in this respect!

2) Since fission is a stochastic process, the actual number of emitted neutrons cannot be determined. There is a distribution in the number of the emitted neutrons (see later). Only the average number of emitted neutrons can be determined. Therefore it is not necessary an integer

We could write more „precisely":

$$^{235}_{92}\mathrm{U} + \mathrm{n} \rightarrow {}^{A_1}_{Z_1}\mathrm{X} + {}^{A_2}_{92-Z_1}\mathrm{Y} + (236 - A_1 - A_2)\cdot \mathrm{n}$$

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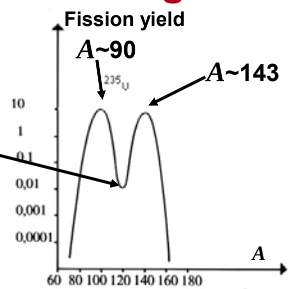
Mass distribution of the fragments

Symmetric fission would be:

$$^{236}_{92}U \rightarrow {}^{118}_{46}Pd + {}^{118}_{46}Pd$$

According to the graph A~120 is ~1000 times less probable



The fission is most probably asymmetric! (^{235}U +n$_{thermal}$)

The mass distribution depends on the fissile material as well as on the energy of the neutrons !

During the fission process

- neutrons are emitted (in average 2,4)
- highly radioactive fission fragments are produced

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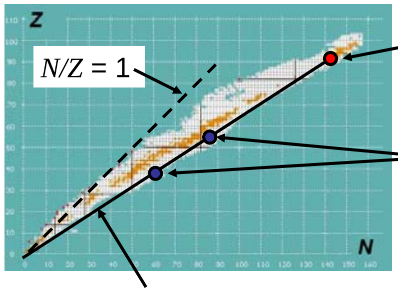
Fission on the (N,Z) „map"

$^{236}_{92}\mathrm{U}_{144}$

$N/Z = 144/92 = 1{,}56$



Starting point: $(N/Z \sim 1{,}56)$ (stable nucleus)

Final points if $N/Z \sim 1{,}56$ remains (asymmetric fission)

$N/Z = 1{,}56$

With respect of stable nuclei NEUTRONRICH nuclei

Highly radioactive (β^- decay) fission products

Neutron emission is necessary ($\bar{\nu}$ ~ 2,4)

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Energy balance of the fission $^{235}U(n,f)$

Energy is released by several processes. This influences the time- and spatial distribution of the heat source

Kinetic energy of the fragments	**168 MeV**	**(82,0 %)**
Energy of the β-particles of the fragments	**8 MeV**	**(3,9 %)**
Total energy of the fission neutrons	**5 MeV**	**(2,4 %)**
Total energy of the prompt γ -rays	**7 MeV**	**(3,4 %)**
Energy of the γ –radiation of the fragments	**7 MeV**	**(3,4 %)**
Energy of the antineutrinos emitted during the β-decay of the fragments	**10 MeV**	**(4,9 %)**
TOTAL	**205 MeV**	**(100%)**

- **Short range (in the fuel and close to it)**
- **Medium range (coolant, reactor vessel, shielding)**
- **Very long range (escapes)**
- **Instantly (prompt)**

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Example: suppose that the fission of ^{236}U occurs the following way: $^{236}U \longrightarrow {}^{90}Kr + {}^{143}Ba + 3n$
Calculate the distance of the two fragments, at the scission point, when only Coulomb-forces act on them! Their total kinetic energy will be 168 MeV.

Solution:
At the scission point they have only Coulomb potential energy, this is transformed to kinetic energy:

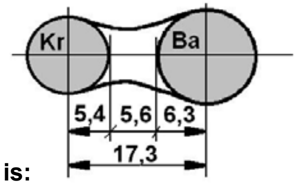
$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Z_1 Z_2 e^2}{d}=168\cdot 1{,}6\cdot 10^{-13}\ \mathrm{J}$$ **Here** $Z_1=36$ **(Kr),** $Z_2=56$ **(Ba)**

From this: $d \sim 17{,}3\ \mathrm{fm}$

Using $R=r_0\sqrt[3]{A}$ **we get for the radius of the two nuclei:**

$R_{Kr}=5{,}4\ \mathrm{fm},\ R_{Ba}=6{,}3\ \mathrm{fm}$

The geometry of the „scission" is:



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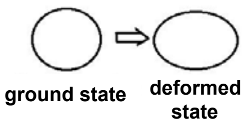
The fission barrier

The fission begins with a small deformation of the nucleus



(deformation parameter: ε)

How much energy is needed?

The energy change is estimated using the Weizsäcker-formula

$$B=\boxed{b_V A}-b_F\cdot A^{2/3}-b_C\cdot\frac{Z^2}{A^{1/3}}-\boxed{b_A\cdot\frac{(N-Z)^2}{A}+b_P\cdot\delta\cdot A^{-3/4}}$$

During the deformation
- **volume = constant (uncompressible nuclear matter)**
- **asymmetry, pairing energy = assumed to be constant**
- **surface ⟶ increases** $b_F A^{2/3} \Rightarrow b_F A^{2/3}(1+a_F\cdot\varepsilon)$
- **Coulomb-energy ⟶ decreases** $b_C\frac{Z^2}{A^{1/3}} \Rightarrow b_C\frac{Z^2}{A^{1/3}}(1-a_C\cdot\varepsilon)$
  (protons get further away)

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The coefficients (according to precise calculations):
$a_F \sim 0{,}025$ **és** $a_C \sim 0{,}012.$

The change in energy: $\Delta E=\left(0{,}025\cdot b_F\cdot A^{2/3}-0{,}012\cdot b_C\frac{Z^2}{A^{1/3}}\right)\cdot\varepsilon$

If $\Delta E<0$, then every small ε deformation decreases the energy, the nucleus spontaneously fissions!

$$\left(0{,}025\cdot b_F\cdot A^{2/3}-0{,}012\cdot b_C\frac{Z^2}{A^{1/3}}\right)<0 \Rightarrow \boxed{\frac{Z^2}{A}\geq\frac{0{,}025\cdot b_F}{0{,}012\cdot b_C}\approx 54}$$

(since $b_F=2{,}85\cdot 10^{-12}$ J, $b_C=0{,}11\cdot 10^{-12}$ J)

For heavy nuclei $Z/A\sim 0{,}39$, therefore for $Z>136$ there is no „activation energy" for the fission. (In fact, already around $Z\sim 110$ the activation energy is so small, that the nuclei spontaneously fission quickly by the quantum-mechanical tunnel effect.)

The Periodical System is finished because of the fission!!

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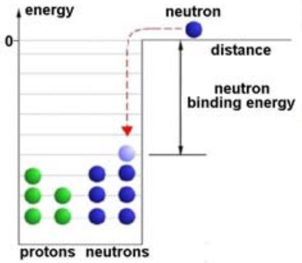
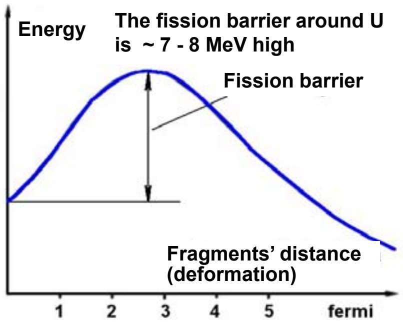
The fission barrier

If $\Delta E > 0$, then small ε deformations need some energy.
This leads to the formation of a fission (energy) barrier.

How can a slow neutron (kinetic energy~0) make ^{235}U fission?





The binding energy of the neutron can make the nucleus climb over the fission barrier!

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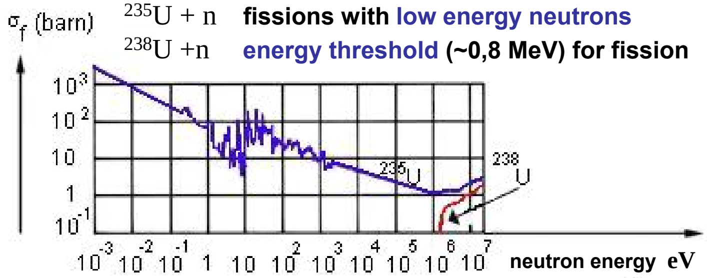
Fission cross sections

^{235}U + n fissions with low energy neutrons
^{238}U + n energy threshold (~0,8 MeV) for fission



^{235}U and ^{238}U fission cross sections in function of the neutron energy

The cause of the difference is in the pairing energy. $^{235}_{92}U_{143}$ is even-odd (less bound), and $^{238}_{92}U_{146}$ is even-even (strongly bound). For the other heavy nuclei in the region the fission cross-sections behave similarly.

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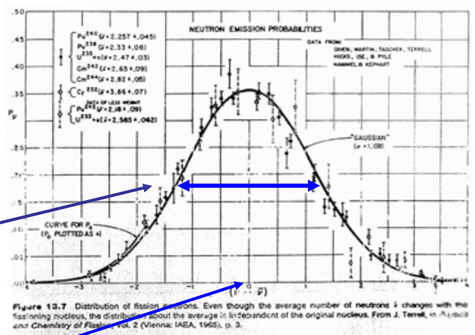
Fission neutrons

The number of emitted neutrons is not constant, it is scattered around a mean value. Can be well approximated by a Gaussian-distribution

The mean value depends on the energy of the incoming neutron and also on the fissile isotope



FWHM: ~ 2,5 (not depending on the fissile isotope)

Mean value: $\bar{\nu} = 2{,}43$ (for ^{235}U+n_{th} fission)

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Time and energy of the fission neutrons (emission time after the fission)

a) Prompt neutrons

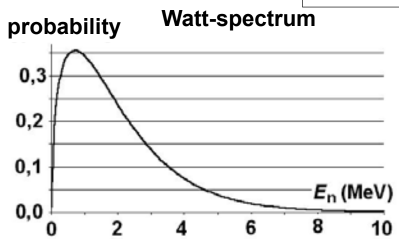
Most of the neutrons are emitted „immediately" at the fission.
These are the prompt neutrons ($t < 10^{-16}$ s).

Their average energy ~ 2 MeV

Energy-distribution: „Watt-spectrum"

$$N(E_n) \sim 0{,}484 \cdot e^{-E_n/E_0} \cdot \sinh\sqrt{2\frac{E_n}{E_0}}$$

$(E_0 = 1\ \text{MeV})$



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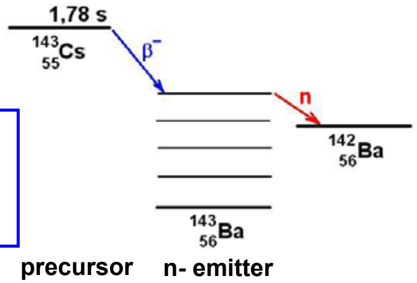
b) Delayed neutrons

The fission fragments necessarily contain more neutrons than the stable isotopes.
Consequently they all undergo β^--decays.

In some cases after the β^--decay also the daughter nucleus can emit neutrons ⟶ delayed neutrons

A possible example:
(The neutron will be emitted with 1,78 s half life!)



There are many similar decay chains, with different half lives (T_i), different yields (β_i)

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Delayed neutrons are grouped into 6 groups (according to their half lives)

	E_n(MeV)	T_i(s)	β_i(%)	Typical precursor
1	0,25	56	0,020	^{87}Br, ^{142}Cs
2	0,56	23	0,143	^{88}Br, ^{137}I
3	0,43	6,2	0,128	^{89}Br, ^{138}I
4	0,62	2,3	0,255	^{94}Kr, ^{139}I, ^{143}Cs
5	0,42	0,6	0,074	^{140}I, ^{145}Cs
6	0,51	0,2	0,030	^{87}As, ^{143}Xe

Total yield: β = 0,65 %
Delayed neutron ratio: $\beta = \dfrac{\text{(delayed n)}}{\text{(total n)}} \sim \dfrac{\text{(delayed n)}}{\text{(prompt n)}}$

Delayed neutrons' emission rate after the fission:

$$N(t) = \sum_{i=1}^{6} \beta_i \cdot e^{-\ln 2 \cdot \frac{t}{T_i}}$$

Their role is very important in the control of the chain reaction!!

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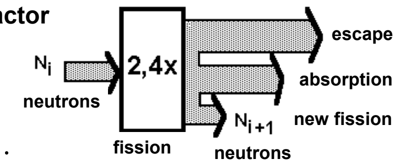
Chain reaction with neutrons

The „neutron-budget"
What can happen with a neutron?
- Escapes from the reactor
- Gets absorbed (n,γ)
- Induces fission (n,f)



Neutron „generations"
$N_1, N_2, N_3, \ldots N_i, N_{i+1}, \ldots$
N_i denotes the number in the i-th generation

Effective neutron multiplication factor: $\boxed{k_{eff} = \dfrac{N_{i+1}}{N_i}}$ (definition)

If
$$\begin{cases} k_{eff} < 1, & \text{chain reaction decreases („subcritical")} \\ k_{eff} = 1, & \text{chain reaction is stationary („critical")} \\ k_{eff} > 1, & \text{chain reaction increases („supercritical")} \end{cases}$$

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Time-behaviour of the chain reaction

Starting point: $\boxed{k_{eff} = \dfrac{N_{i+1}}{N_i}}$

A bit of algebra: $k_{eff} - 1 = \dfrac{N_{i+1}}{N_i} - 1 = \dfrac{N_{i+1} - N_i}{N_i} = \dfrac{\Delta N}{N_i}$

The time between two generations: $\Delta t = \ell$ (generation time)

Multiplying the two equations: $\Delta t \cdot \left(k_{eff} - 1\right) = \ell \cdot \dfrac{\Delta N}{N}$

From this we get: $\dfrac{\Delta N}{\Delta t} = \left(\dfrac{k_{eff} - 1}{\ell}\right) \cdot N$

After the $\Delta t \to 0$ limes $\dfrac{dN}{dt} = \left(\dfrac{k_{eff} - 1}{\ell}\right) \cdot N(t)$

The solution of this differential equation:

$$\boxed{N(t) = N_0 \cdot e^{\left(\frac{k_{eff} - 1}{\ell}\right) t}}$$

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Obviously, for $k_{eff} = 1 \Rightarrow N(t)=N_0=\text{const}$

$$N(t) = N_0 \cdot e^{\left(\frac{k_{eff}-1}{\ell}\right)t}$$

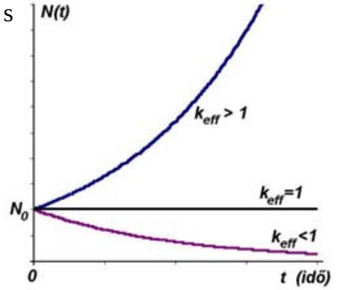
For $k_{eff} > 1 \Rightarrow N(t)$ increases exponentially

For $k_{eff} < 1 \Rightarrow N(t)$ decreases exponentially.

The changing rate is determined by $\left(\frac{k_{eff}-1}{\ell}\right)$

For prompt neutrons

the generation time $\ell \sim 10^{-4}$ s



__Example__:

Suppose, that k_{eff}=1,001

The change in the number of neutrons (and in the reactor power) during 1 s:

$$N(1) = N_0 \cdot e^{\frac{1{,}001-1}{0{,}0001}\cdot 1} =$$

$$= N_0 \cdot e^{10} = 22026 \cdot N_0$$

It CAN NOT be controlled! (prompt-critical)

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The role of the delayed neutrons

The generation-time for the delayed neutrons is much larger, because of the half-life of the β-decay of the precursor. It can be several secundum!

The very important role of the delayed neutrons:
They increase the effective generation time!

The system can be controlled only, if the condition of prompt-criticality is not present.
This means that $k_{eff}<1$ without the delayed neutrons!

Therefore it is absolutely necessary, that $k_{eff} < 1+\beta = 1{,}0065$ should be fulfilled any time, at any operational state of a nuclear reactor!

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The reactivity and its unit

__The definition of reactivity:__ $\rho = \frac{k_{eff}-1}{k_{eff}}$

For a prompt-critical system: $k_{eff} = 1+\beta$,

therefore its reactivity: $\rho = \frac{1+\beta-1}{1+\beta} \approx \beta$

(Since $1+\beta = 1{,}0065 \sim 1$)

A practical unit of the reactivity is expressed by its ratio to the delayed neutron fraction. It is denoted by \$ (dollar).

The reactivity is 1 \$, if $\frac{\rho}{\beta} = 1$

A 1\$ reactivity system is prompt critical. Therefore the actual reactivity is expressed usually in cents (¢).

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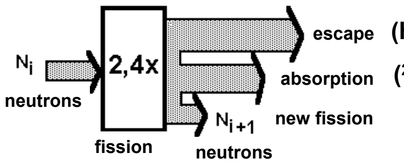
Options to realise the nuclear chain reaction

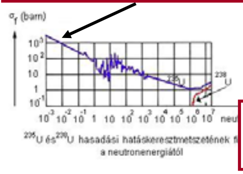


escape (happens on the surface)

absorption (^{238}U absorbs only, does not fission)

The ratio of the „new fission" should be increased.
There are several ways to do that

Slow down the neutrons (use moderator) (fission cross-section increases)



Decrease the ratio of the absorption

Increase the ratio of ^{235}U/^{238}U (enrichment)

Decrease the ratio of the „escape"

Large size (surface/volume) decreases Critical mass

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Moderator properties and materials

A material is appropriate for slowing down neutrons, if

- it has small A (large energy-transfer in one collision),
- it has large neutron-scattering cross-section,
- it has small neutron-absorption cross-section.

A material with these properties is called moderator.

Best moderator is heavy water and pure graphite

In light water the hydrogen absorbs neutrons by neutron capture: $^{1}H + n \longrightarrow {}^{2}H + \gamma$

The possible realisations of self-sustaining chain reaction

Enrichment of fuel	*Moderator material*
Natural uranium (0,71% ^{235}U)	Heavy water, pure graphite
3-5% enriched uranium	Light (natural) water
>40% enriched uranium (>90%)	No need for a moderator (nuclear weapon)

Remember: the moderator HELPS the chain-reaction!

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Self-test questions (cont.)

7. The height of the fission barrier is about 7-8 MeV for the uranium isotopes. How is it possible that a thermal neutron (energy ~0,03 eV) can induce fission in ^{235}U?
8. Why can a thermal neutron induce fission in ^{235}U, and can not induce fission in ^{238}U, if the fission barrier is about the same height for both isotopes?
9. How does the fission cross section depend on the neutron velocity for very slow neutrons, and for the ^{235}U(n,f) reaction?
10. What kind of distribution describes the number of emitted neutrons? What is the mean value for ^{235}U(n_{th},f)
11. How are the delayed neutrons produced? How much is their proportion? What determines their „delay”?
12. What may happen to a neutron in a reactor? Which process is important to maintain a chain reaction?
13. How is k_{eff} defined? What is its relation to the behaviour of the chain reaction?

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Self-test questions (cont.)

14. What is the role of the delayed neutrons in the chain reaction?
15. What is prompt-criticality? What condition must be fulfilled to avoid it?
16. How is the reactivity defined? What is its unit?
17. Can the reactivity be negative? What does that mean?
18. What kind of properties should have a good moderator material?
19. What kind of effect has the moderator on the chain reaction?
20. What are the options that can be used (alone or combined) to increase the ratio of „new fission” for obtaining a self-sustaining chain reaction?
21. By what combinations of enrichment and moderator are possible to realise a self-sustaining chain reaction?

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