HUVINETT

Basic Properties of Nuclei

<u>Course on Nuclear Fundamentals</u>
1st lecture

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The composition of the nucleus

Z protons, ('atomic number') } **nucleons**
N neutrons
$A = Z + N$ (mass-number, number of nucleons)

Notation: $^{A}_{Z}\mathrm{X}_{N}$ **e.g.** $^{4}_{2}\mathrm{He}_{2}$ $^{40}_{19}\mathrm{K}_{21}$ $^{238}_{92}\mathrm{U}_{146}$

Redundant, $^{238}\mathrm{U}$ alone is sufficient !

	proton	neutron
mass	1,67265·10^{-27} kg	1,67495 ·10^{-27} kg
charge	+e	0

<u>Notations</u>:
- **nuclei with the same number of protons (Z) : ISOTOPES**
- **nuclei with the same number of nucleons (A) : ISOBARS**
- **nuclei with the same number of neutrons (N): ISOTONES (rarely used)**

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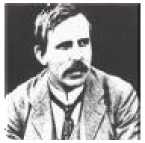
The size of the nucleus

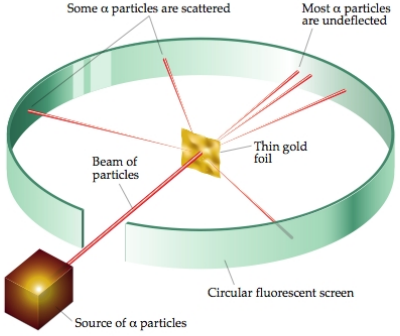


1911: Rutherford experiment

- **α– particles scattered on thin gold (Au) foil**

Why gold?
- **VERY thin foils can be prepared from gold! (few atomic layer only)**

The experimental apparatus should be in vacuum



Source: http://session.masteringchemistry.com/problemAsset/1070873/24/BLB-1070873-Rutherford_v2.jpg

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What was expected?
The Coulomb interaction between the positively charged part of the atom and the alpha-particle: a „Coulomb-hill" to climb.

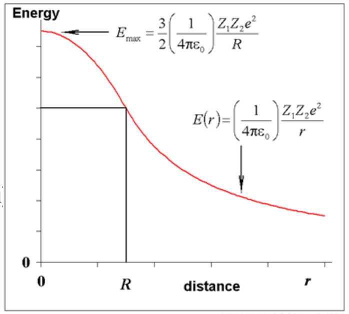
The maximal height of the „Coulomb hill":

$$E_{\max} = \frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Z_1 Z_2 e^2}{R}$$

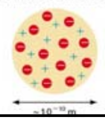
$Z_1 = 2$ (atomic number of He)
$Z_2 = 79$ (atomic number of Au)

$$\left(\frac{1}{4\pi\varepsilon_0}\right) = 9\cdot10^9 \text{ J}\cdot\text{m/Cb}^2$$

$e = 1{,}6 \cdot 10^{-19}$ Cb



Thomson's atomic model



„Thomson's-atomic model" (pudding model): the radius of the positively charged part is approx. the radius of the atom ~ 10^{-10} m

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If $R = R_{atom} \sim 10^{-10}$ m
then $E_{max} \sim$ 5,5 $\cdot 10^{-16}$ J
But: $E_{alpha} \sim 7700{,}0 \cdot 10^{-16}$ J

Like cannon ball through a paper!

http://www.reak.bme.hu/fileadmin/user_upload/felhasznalok/sukosd/letoltesek/Rutherford_en/Rutherford.htm

Experimental result:
There were also „back-scattered" particles!

Conclusion:
The potential hill should be „higher", than the energy of the α-particle!
That is: $\frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Z_1 Z_2 e^2}{R} > E_{alpha}$

From this we have: $R < \frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Z_1 Z_2 e^2}{E_{alpha}}$

Using $E_{alpha} \sim 7700 \cdot 10^{-16}$ J we calculate: $R < 10^{-14}$ m, which is about ten-thousand times smaller than the size of the atoms!
The mass and the positive electric charge is concentrated in the very small atomic nucleus!

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Hofstädter experiment (1950-54) :
Scattering of high-energy electrons on different materials
Even the charge distribution could be determined

Why was high-energy (~300 MeV) electrons needed?
Electrons are even not repelled by the nucleus!!!

Remember: the resolving power of a microscope depends on the wave-length (λ) used!
The wave-length of a particle (de Broglie): $\lambda = \frac{h}{p} = \frac{h}{m v}$

Resolving (spatially) small details, small wave-length is needed: large velocity, i.e. large energy.

Rutherford was lucky: because of the much larger mass of the alpha-particle, its wave-length was small enough to discover the atomic nucleus!

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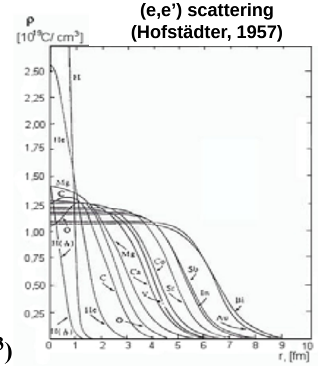
Results of the Hofstädter experiment:
- The central density ~ constant
- $R = r_0 \cdot A^{1/3}$, where
- A = number of nucleons (mass-number)
- $r_0 = 1{,}07\cdot10^{-15}$ m = 1,07 fm.

Charge distribution from (e,e') scattering (Hofstädter, 1957)



The charge-density can be well described by a Fermi-function:

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{d}}}$$

where
R is the nuclear radius, ($\sim r_0 \cdot A^{1/3}$)
d is the surface „diffusity" (~constant)

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The mass of the nucleus

The „mass-defect": $M(Z,A) = Z \cdot m_{proton} + (A-Z) \cdot m_{neutron} - \Delta M$

Cause: the nucleus is a bound system, energy is needed to take it apart.

Einstein's relation: $E = m \cdot c^2$. If energy is needed, then also mass is needed to take the bound nucleons apart!

With measuring the mass-defect (precise measurement of nuclear mass) the binding energy of the nucleus can be determined: $B = \Delta M \cdot c^2$

Measuring the mass of the atoms:

With mass-spectrometers (mass-spectroscopes)

- The atoms get first ionised,
- The ions get accelerated with electrical fields
- The accelerated ions will be deflected by electric and magnetic fields
- The mass can be determined from the deflection of ions

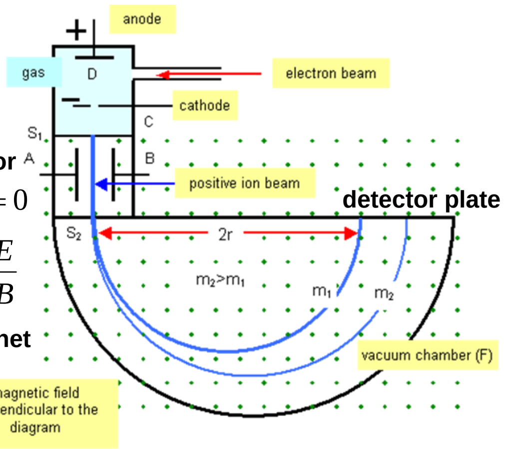
Aston's mass spectrometer (1919)

ion source & acceleration

velocity selector $q\mathbf{E} - q[\mathbf{v} \times \mathbf{B}] = 0$

from this: $v = \frac{E}{B}$

analysing magnet



Source: http://www.schoolphysics.co.uk/age16-19/Atomic%20physics/Atomic%20structure%20and%20ions/text/Mass_spectrometer/images/1.gif

The principle of the analyser:

$$\frac{mv^2}{r} = q \cdot v \cdot B$$

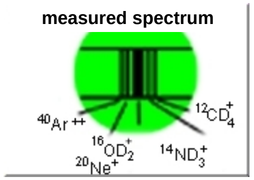
centripetal force = Lorentz force

From this we get: $r = \left(\frac{m}{q}\right) \cdot \frac{v}{B}$

For example: $(m/q)=20$ „mixed" beam:
$^{40}Ar^{++}$, $^{20}Ne^{+}$, $^{16}OD_2^{+}$, $^{14}ND_3^{+}$, $^{12}CD_4^{+}$

The spectrometers select according to (m/q)

Small mass differences can be measured very precisely!



The nuclear mass is often expressed in atomic mass unit:

1 u = $\frac{M(^{12}C)}{12}$

The mass $M(^{12}C)$ here is the *atomic mass* !!! (mass of 6 electrons included)

1 u = 1,66043(2)·10^{-27} kg = 931, 478 MeV/c^2

Why was the ^{12}C atom chosen?
Because carbon atoms can form a huge variety of molecules with different molecular weights!

Mass-doublet method:
For example: $C_{10}H_8$ (naphthalene) and C_9H_{20} (nonane) both have 128 nucleons. However, their masses are a bit different! $\Delta = m(C_9H_{20}) - m(C_{10}H_8) = 12 \cdot m(^1H) - m(^{12}C)$

A good measurement gives: $\Delta = 0{,}09390032 \pm 0{,}00000012$ u

From this we get:

$$m(^1H) = \frac{m(^{12}C)}{12} + \frac{\Delta}{12} = 1{,}00782503 \pm 0{,}00000001 \text{ u}$$

Very precise!!

Binding energy of the nucleus

The **binding energy**: $B = \Delta M \cdot c^2$ (Einstein)

By measuring the mass-defect (mass-spectrometers) the binding energy can be determined

<u>Energy and binding energy</u>:

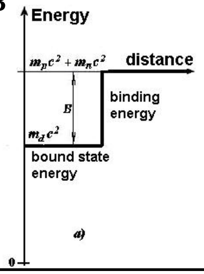
Einstein: $E = m \cdot c^2$. Since $m \geq 0$, the total **energy** is $E \geq 0$.

Example: look at the mass of deuteron (^{2}H) and its energy!

$m_d = m_p + m_n - \Delta M$ (multiply by c^2)

$m_d c^2 = m_p c^2 + m_n c^2 - B$

Usually the zero point of the energy axis will be chosen at the unbound system (right side of picture). If so, the energy of the bound system will be NEGATIVE: $E = -B$



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Binding energy of nuclei

(Semi-empirical binding energy formula of Weizsäcker)

Starting point: nuclear density ~ constant, thus the nucleus is like an (electrically charged) liquid drop (Liquid drop model)

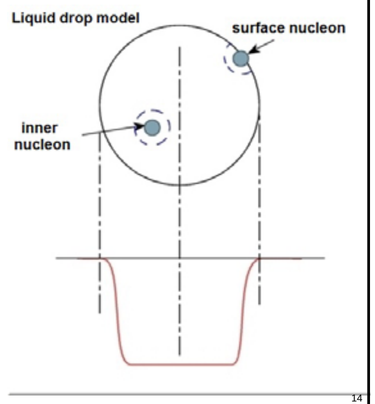
The nucleons interact only with neighbors. If all nucleon was „inner" one, then the total binding energy would be $B = b_V \cdot A$.

(b_V is the binding energy of one „inner" nucleon)

The „surface" nucleons are bound weaker, thus

$B = b_V \cdot A - \beta \cdot 4\pi R^2$

Here β is a constant.



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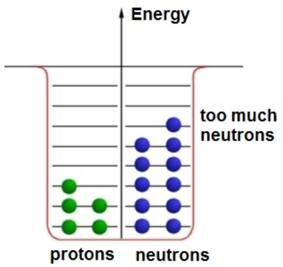
So far only the nuclear interaction was taken into account. The nucleus has also Ze electric charge, and it makes the binding weaker (because of the Coulomb-energy due to the mutual repel of the protons):

$$B = b_V A - \beta \cdot 4\pi R^2 - \frac{3}{5} k \frac{Z^2 e^2}{R}$$

Because of quantum mechanics the Pauli-principle is valid for the protons and the neutrons (at most 2 particles can be on an energy level). Too much neutron or proton (asymmetry) weakens the binding:



$$B = b_V A - \beta \cdot 4\pi R^2 - \frac{3}{5} k \frac{Z^2 e^2}{R} - b_A \frac{(N-Z)^2}{A}$$

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Finally: empirical fact is that nuclei are stronger bound, if their number of protons or neutrons (or both) are even (pairing energy).

$$B = b_V A - \beta \cdot 4\pi R^2 - \frac{3}{5} k \frac{Z^2 e^2}{R} - b_A \frac{(N-Z)^2}{A} + b_P \delta \cdot A^{-3/4}$$

Here $\delta = 1$, if the nucleus is even-even

$\delta = 0$, if the nucleus is even-odd

$\delta = -1$, if the nucleus is odd-odd

Use now the relation $R = r_0 \cdot A^{1/3}$, and unify the different constants to one constant at every member:

$$B = b_V A - b_F \cdot A^{2/3} - b_C \cdot \frac{Z^2}{A^{1/3}} - b_A \cdot \frac{(N-Z)^2}{A} + b_P \cdot \delta \cdot A^{-3/4}$$

This is the semi-empirical binding energy formula of Weizsäcker

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The name of the different members in the formula
(the value of the constants are in brackets)

- volume energy (b_V=2,52·10^{-12} J)
- surface energy (b_F=2,85 ·10^{-12} J)
- Coulomb-energy (b_C=0,11 ·10^{-12} J)
- Asymmetry energy (b_A=3,80 ·10^{-12} J)
- Pairing energy (b_P=1,49 ·10^{-12} J)

These constants were determined empirically. With these 5 constants the binding energy of the more than 2000 nuclei can be well described with a precision of 1-2 %

Average binding energy of one single nucleon: $b = B/A$.

How „deep" is a nucleon in average inside the nucleus?
How much is the average energy of one single nucleon?
$\varepsilon = -b = -B/A$.

Importance: during spontaneous processes ε decreases
(energy minimum principle)

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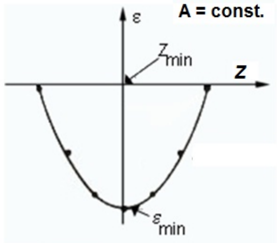
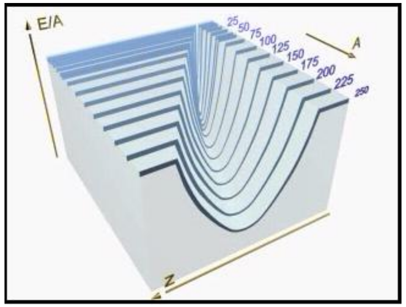
Since B is a function of the nuclear composition (Z,A), thus ε is a function of those as well.
$\varepsilon = \varepsilon(Z,A)$. This can be drawn as a „surface".

$$\varepsilon = \frac{E}{A} = -\frac{B}{A} = -\frac{1}{A}\left(b_V A - b_F \cdot A^{2/3} - b_C \cdot \frac{Z^2}{A^{1/3}} - b_A \cdot \frac{(N-Z)^2}{A} + b_P \cdot \delta \cdot A^{-3/4}\right)$$

Note, that the A = const. cuts are parables!

$$\varepsilon(Z)\big|_{A=const.} = a \cdot (Z - Z_{min})^2 + \varepsilon_{min}$$





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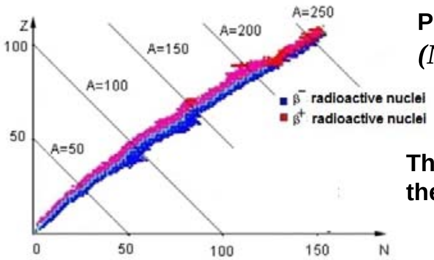
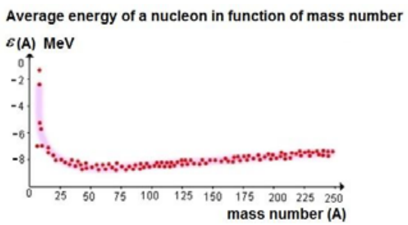


Position of Z_{min} on the (N,Z) „map"

This helps to understand the radioactive decays!

ε_{min} in function of the mass-number (A)

This helps to understand the energy production from the nuclei (nuclear energy production)



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Excited states, energy levels

Nuclei are quantum mechanical systems:
they have discreet energy states, with well defined quantum numbers (angular momentum, parity, etc.).

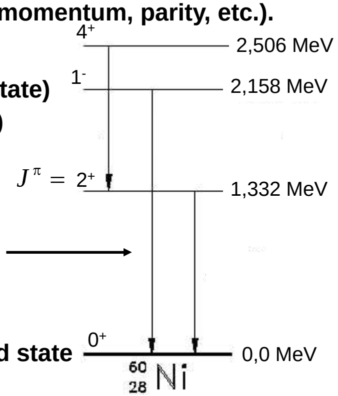
Properties of a state:

- Energy (above the ground state)
- Total angular momentum (J)
- Parity (π) can be + or –
- Half-life (T)

Gamma-transitions can occur between the energy levels

$$h\nu = E_\gamma = E_1 - E_2$$

Ground state



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Self-test questions

1. Select out from the following nuclei
$^{45}_{20}\text{Ca}$, $^{45}_{21}\text{Sc}$, $^{40}_{20}\text{Ca}$, $^{45}_{22}\text{Ti}$, $^{45}_{23}\text{V}$, $^{44}_{20}\text{Ca}$, $^{44}_{24}\text{Cr}$
   - the isobars
   - the isotopes
2. Why should the apparatus be in vacuum at the Rutherford-experiment?
3. Why is different the „Coulomb-hill" inside the positively charged part from the outside part at the Rutherford experiment?
4. Hofstädter needed electrons of 300 MeV to determine the size of the nucleus. Why were ~ 5 MeV alpha-particles enough for Rutherford?

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Self-test questions (cont.)

5. Why is a velocity selector needed in Aston's mass-spectrograph before the analysing magnetic deflection?
6. Where would the naphthalene and nonane ions hit in a mass-spectrograph, if there was no binding energy in the nuclei, and if the mass of the neutron and proton was the same?
7. What is the difference between energy and binding energy?
8. What are the main assumptions of the liquid drop model? What do we learn about the interaction between the nucleons from this model?
9. A liquid drop is hold <u>stronger</u> together because of the surface tension of the liquid. Why do we say then, that the surface energy <u>weakens</u> the binding of the nuclei?
10. For A=const. the average energy of a nucleon (ε) is described by 3 parameters of a parable: a, Z_{min}, and ε_{min}. Derive the A-dependence of these parameters from the Weizsäcker-formula!

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Self-test questions (cont.)

11. Why is the average energy of a nucleon (ε) an important parameter?
12. Which nuclear energy level is described by the Weizsäcker formula?
13. How would look the $Z_{\text{min}}(A)$ and the $\varepsilon_{\text{min}}(A)$ functions, if there was no Coulomb-repulsion? (all other terms would remain in the Weizsäcker formula)
14. What parameters are usually used to characterise a nuclear energy level?
15. What are the possible values of the parity of a level?
16. Does the „pairing energy" in the Weizsäcker formula has something to do with the parity? Clarify both!
17. What kind of radiation is emitted when a nucleus decays from an excited state to a lower lying state?

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