Radioactivity

Nuclear and Reactor Physics Fundamentals
(BMETE80MX00)

Dr. Csaba Sükösd
honorary professor
sukosd@reak.bme.hu

Budapest University of Technology and Economics
Institute of Nuclear Techniques (BME NTI)

Contents

Alpha, beta, gamma-decay

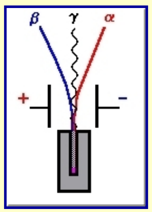
Reminder: the average energy of a nucleon: $\varepsilon = \frac{E}{A} = -\frac{B}{A}$

Radioactive decay: **spontaneous** process
The energy released $\Delta E = \Delta M \cdot c^2$ (ΔM is the „mass-defect")
It can only occur if $\Delta E > 0$ (energy must be released)

In most cases it can be simplified to: $\varepsilon_{\text{initial}} > \varepsilon_{\text{final}}$
(however, there are a few exceptions: e.g. $^3H \rightarrow ^3He$, or $^{14}C \rightarrow ^{14}N$)

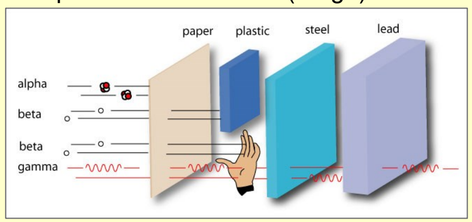
From naturally radioactive substances
3 types of particles are emitted:

- α – particles: ^4_2He nuclei
- β – particles: high energy electrons (positrons)
- γ - radiation: electromagnetic (photons)



Alpha, beta, gamma-decay

The penetration distance (range) are different:



This is important from several practical points of view:

- **Shielding** (designing radiation shields for nucl. installations)
- **Radiation protection** (protection measures)
- **Health effects** (external or incorporated isotopes)
- **Detection of radiation** (e.g. detector material & thickness)

During radioactive decays nuclei transform into each-other

The following terms are used:
parent nucleus decays into **daughter** nucleus

Conserved physical quantities

- **energy** (taking $E = mc^2$ into account)
- **mass number** (number of nucleons, denoted usually: A)
- **electric charge** (nucleus: $+Ze$, electron: $-e$, positron: $+e$)
- **lepton number** (number of „light" particles: electron, neutrino)
- several others (momentum, angular momentum, parity, etc.)

We will consider only the four most important quantities for the 3 decay types

Gamma-decay

The gamma-decay: between energy levels of the nucleus
The **composition does not change**
(mass-number, lepton-charge and electric charge conservation is automatically fulfilled):

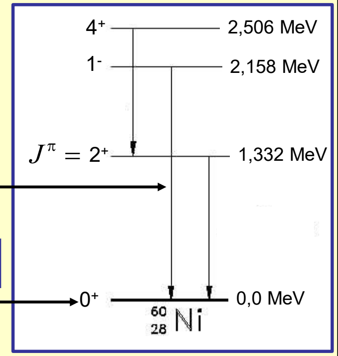
$${}^A_Z X^* \rightarrow {}^A_Z X + \gamma$$

Energy conservation:

Gamma-transitions occur between the energy levels

Photon energy: $h \cdot f = E_\gamma = E_1 - E_2$

Ground state



Gamma-decay (energy)

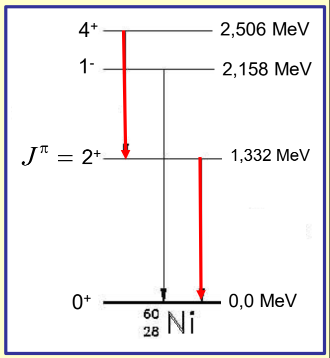
Every isotope has **different** level scheme (like fingerprints)

The possible **energies** of the emitted γ-rays are different (depends also where we start)

For example: 2 γ-energies, if we start from the 4⁺ state:

$$E_\gamma(1) = 2{,}506 - 1{,}332 = 1{,}174 \text{ MeV}$$

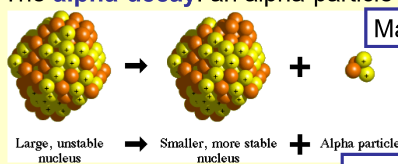
$$E_\gamma(2) = 1{,}332 - 0{,}0 = 1{,}332 \text{ MeV}$$



By finding these γ-rays from a sample in a measurement, we know that ^{60}Ni 4⁺ state was formed ➡ possibility of **analysis of the composition**!

Alpha-decay

The **alpha-decay**: an alpha-particle (He nuclei) is emitted:



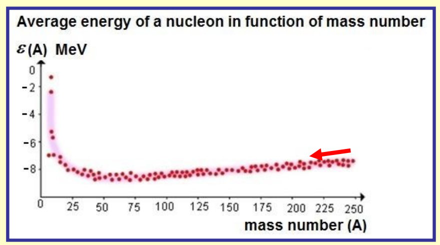
Mass-number conservation

$${}^A_Z X \rightarrow {}^{A-4}_{Z-2} Y + {}^4_2 He$$

Electric charge conservation

No lepton is emitted

Occurs only for heavy nuclei, because only for them will be energy released, if the nucleus gets smaller!



Alpha-decay (energy)

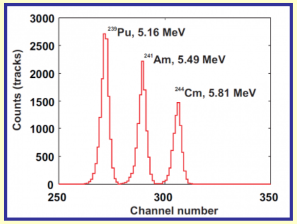
$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$$

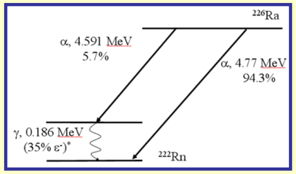
The energy released:

$$Q = \left(M(\mathrm{X}) - M(\mathrm{Y}) - M(\mathrm{He})\right)\cdot c^2 = \Delta M \cdot c^2$$

The Q energy appears in the form of kinetic energy, and is shared between the daughter nucleus (Y) and the He. (Also the momentum conservation law must be considered.)

The alpha spectrum contains lines with well defined energies





Suitable for **analysis**!

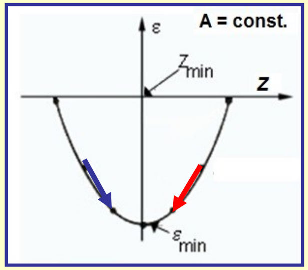
Beta-decays

Light particles (leptons) are emitted.

Mass-number (A) does not change (isobar transitions)

Mass-number is conserved ➡ only **light** particles emitted

From the Weizsäcker-formula we get ($A = const.$ case)



During beta-decays the atomic number (Z) changes by ±1.

$\Delta Z = +1$

$\Delta Z = -1$

Electric charge conservation: since Z changes, the emitted (light) particles should carry electric charge

Three types of beta-decay:

Negative beta-decay (Z increasing) $\quad {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + e^- + \tilde{\nu}$

Positive beta-decay (Z decreasing) $\quad {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + e^+ + \nu$

Electron capture (Z decreasing) $\quad {}^{A}_{Z}\mathrm{X} + e^- \rightarrow {}^{A}_{Z-1}\mathrm{Y} + \nu$

Here e^-: electron, e^+: positron, ν: neutrino, $\tilde{\nu}$: antineutrino

Lepton-number conservation: need of neutrinos/antineutrinos

Released energy is shared **randomly** between the electron (positron) and the antineutrino (neutrino) } Energy spectrum of the electron (positron) is continuous!

Negative beta-decay ($Q > 0$): when too much **neutrons**

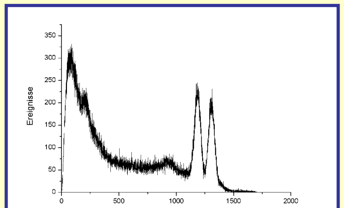
Positive beta-decay ($Q > 2m_e c^2$): } when too much **protons**

Electron capture ($Q > 0$): }

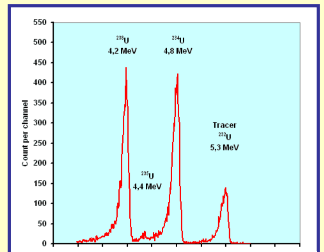
The energy spectrum:

Alpha-decay: } discreet energies

Gamma-decay: }



gamma-spectrum

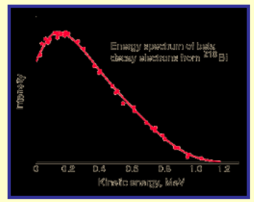


alpha-spectrum

Beta-decay: continuous energy distribution

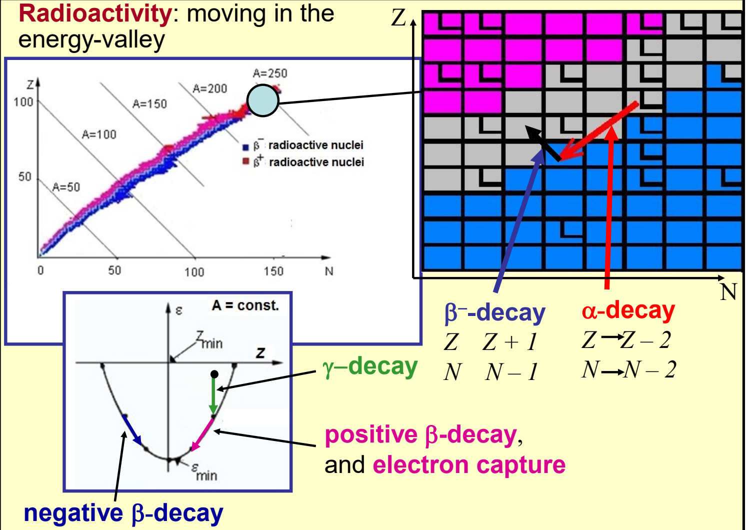
Only endpoint energy is characteristic

β-spectra of different nuclei overlap!

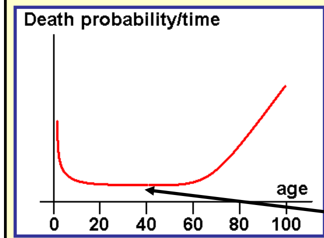


Not suitable to perform analysis!

Radioactivity: moving in the energy-valley



Exponential decay law, half life, activity



The „decay" probability/time for humans change with the age (time of existence)

Death probability = $-\Delta N/N(t)$ here $N(t)$= number of population in a certain age group

It is almost constant for young people (>0 accidents, etc.)

Nuclei are forever „young"!

Decay probability/time = λ = constant [1/s]

$$\frac{dp}{dt} = \frac{-\left(dN/N(t)\right)}{dt} = \lambda \Rightarrow \frac{dN}{dt} = -\lambda \cdot N(t) \Rightarrow \boxed{N(t) = N_0 \cdot e^{-\lambda \cdot t}}$$

Exponential decay law, half life, activity (contd.)

Half life:

The T time, during which the number of nuclei decreases to half of the initial value.

$$N(T) = N_0 \cdot e^{-\lambda T} = \frac{N_0}{2}$$

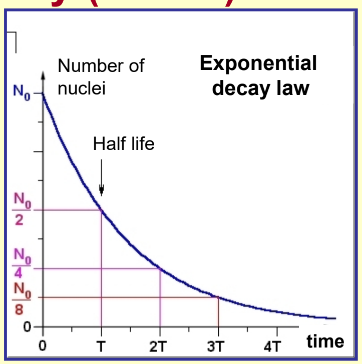
From the second equation:

$$e^{\lambda T} = 2$$

Taking the logarithm: $\boxed{T = \frac{\ln 2}{\lambda}}$

Activity:

Number of decays during unit time: $A = -\frac{dN}{dt}$



Unit of activity: 1 Bq = 1/s (Becquerel)

Using the previous equation we get: $\boxed{A(t) = \lambda \cdot N(t)}$

Poisson-distribution

The radioactive decay is a stochastic (statistical) process!
(described by the decay probability/time: λ)

- The exact decay time of a single atom cannot be predicted
- The exponential decay law can be used only for large number of particles.

The **Poisson-distribution** gives the probability that

- exactly k decays occur
- in a source of a activity
- during t time.

$$P(k, at) = \frac{(at)^k}{k!} e^{-at}$$

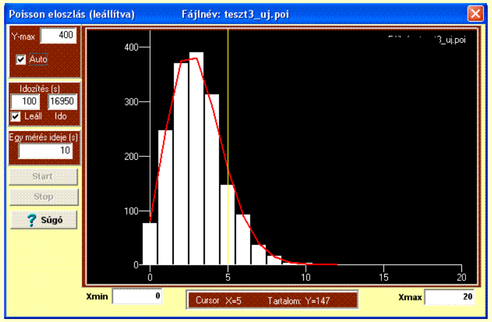
Can be used only if the a activity can be considered constant ($t << T_{half}$).

Poisson-distribution (contd.)

$$P(k,at)=\frac{(at)^k}{k!}e^{-at}$$

expectation value of k: $<k> = a\cdot t$

scatter of k: $\sigma_k = \sqrt{a\cdot t}$



Please note:

$$\sigma_k = \sqrt{<k>}$$

Example:

If N = 100 decays are expected, then the number of decays will be between 90 and 110 in most cases.

Radioactive decay chains

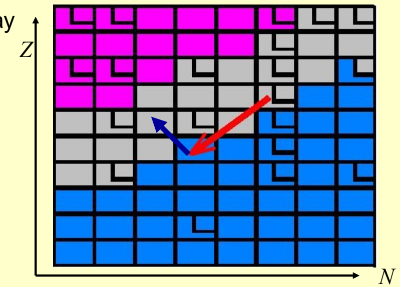
If heavy nuclei (with large A) are decaying, in many cases also the daughter nuclei are radioactive. They decay further, until finally the chain terminates by reaching a stable nucleus.

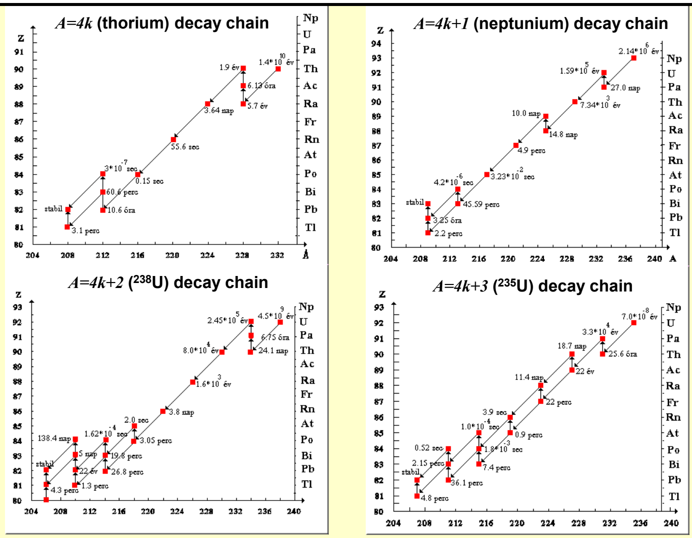
Only alpha-decay changes the mass-number, and always decreases it by 4 (four).

Consequence: the mass-number of every member of the same chain gives the same remainder if divided by 4.

Therefore there are only 4 decay chains:
$A = 4k$, $A = 4k+1$,
$A = 4k+2$, $A = 4k+3$

The α-decays are followed by β-decays (and γ-decays), so that the chain can follow the incline of the energy valley.





Radioactive equilibrium

Consider a radioactive „family" consisting only from 3 members: 1 ⟶ 2 ⟶ 3, and denote the decay constants by λ_1 and λ_2.
The number of the nuclei: $N_1(t)$, $N_2(t)$, $N_3(t)$.

The equations describing the change in the number of nuclei:

$$\frac{dN_1}{dt} = -\lambda_1 \cdot N_1(t)$$ (only decays)

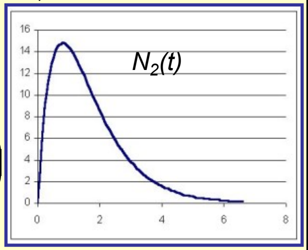
$$\frac{dN_2}{dt} = +\lambda_1 \cdot N_1(t) - \lambda_2 \cdot N_2(t)$$ (generated from the previous and also decays)

$$\frac{dN_3}{dt} = +\lambda_2 \cdot N_2$$ (only generated from the previous)

The solution of the first equation is already familiar:

$$N_1(t) = N_1(0)\cdot e^{-\lambda_1 \cdot t}$$

Initial conditions: $N_2(0) = 0$ and $N_3(0) = 0$, i.e. initially we have no „2" and „3" material, only material „1".



The solution for $N_2(t)$:

$$N_2(t) = N_1(0)\frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

(if $\lambda_1 \neq \lambda_2$).

The activity of the „2" isotope:

$$a_2(t) = \lambda_2 \cdot N_2(t) = a_1(0)\frac{\lambda_2}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Using, that $a_1(t) = a_1(0)\cdot e^{-\lambda_1 t}$ we get:

$$a_2(t) = a_1(t)\frac{\lambda_2}{\lambda_2-\lambda_1}\left(1-e^{-(\lambda_2-\lambda_1)t}\right)$$

Special cases: $a_2(t) = a_1(t)\frac{\lambda_2}{\lambda_2-\lambda_1}\left(1-e^{-(\lambda_2-\lambda_1)t}\right)$

1) If $\lambda_2 > \lambda_1$, then after long enough time the exponent gets negligibly small, and: $a_2(t) \approx a_1(t)\frac{\lambda_2}{\lambda_2-\lambda_1}$

From this we get: Independent of time! $\frac{a_2(t)}{a_1(t)} = \frac{\lambda_2}{\lambda_2-\lambda_1} = \text{konst.}$

This is called **transient** equilibrium.

2) If $\lambda_2 >> \lambda_1$, then the condition of transient equilibrium is met, but additionally, λ_1 can be neglected in the denominator, and we get: $\frac{a_2(t)}{a_1(t)} = \frac{\lambda_2}{\lambda_2} = 1$ With other words: $a_1(t) = a_2(t)$

It can be shown similarly, that if λ_1 is much smaller than all other decay constants, after long enough time we get $a_1(t) = a_2(t) = a_3(t) = \ldots.$, in a many-member decay chain.

This is called **secular** equilibrium.

In secular equilibrium: $a_1(t) = a_2(t) = a_3(t) = \ldots$

Using $a(t) = \lambda \cdot N(t) = \frac{N(t)}{T}\ln 2$ we get:

$$\frac{N_1(t)}{T_1} = \frac{N_2(t)}{T_2} = \frac{N_3(t)}{T_3} = \ldots$$

This can be written in a different way:

$$N_1(t) : N_2(t) : N_3(t) \ldots = T_1 : T_2 : T_3 : \ldots$$

In secular equilibrium, the ratio of the quantities (number of atoms) of the members equals to the ratio of their half-lives.

Practical use: this makes possible the determination of very long half-lives (For example: the half life of ^{238}U is 4,5 billion years.)

Radioactive dating

Using the decay properties of a radioactive substance one can conclude about the age of the sample

Problem: the initial conditions should be known!

Most commonly used isotopes for radioactive dating :

Isotope	Half life	Abundance
^{3}H (tritium)	12,262 year	$1 \cdot 10^{-18}$
^{14}C (radiocarbon)	5568 year	$2 \cdot 10^{-12}$
^{40}K	$1,3 \cdot 10^{9}$ year	$1,19 \cdot 10^{-4}$
^{87}Rb	$50 \cdot 10^{9}$ year	0,278
^{238}U	$4,51 \cdot 10^{9}$ year	0,992739
^{235}U	$0,704 \cdot 10^{9}$ year	0.007204
^{232}Th	$13,9 \cdot 10^{9}$ year	1.0

For most precise dating the half-life of the isotope should be close to the age of the sample (at least similar orders of magnitude)

Geological dating (10 million years – few billion years)
- relative
- absolute

Relative geological dating (non-nuclear methods)
- paleontological (fossils in sedimentary rocks)
- based on the position in geological profile

Absolute dating (nuclear methods)
- Rubidium-strontium (Rb-Sr) method
- Lead-helium method (Th, or uranium decay chains)
- Potassium-argon method (K-Ar)

Rubidium-strontium method: $^{87}Rb \xrightarrow[\beta^-]{50 \text{ billion y}} {}^{87}Sr$

The age of the rock can be determined by the ratio: $\frac{^{87}Sr}{^{87}Rb}$

Lead-helium method: based on the radioactive decay chains.
- From ^{238}U we get finally ^{206}Pb. In the chain: 8 α-decays.
- From ^{232}Th we get ^{208}Pb. In the chain: 6 α-decays.
- From ^{235}U we get ^{207}Pb. In the chain: 7 α-decays

Therefore helium and lead accumulate in the rock.

Difficulties:
- difficult to separate the lead-isotopes
- usually all the three decay chains are present
- the helium is a noble gas. Some quantity can escape
- all chains contain an isotope of Rn (radon) this is also a noble gas, can escape, the chain gets „broken"

Potassium-argon method (T = 1,3 billion years)

$^{40}K \longrightarrow {}^{40}Ca$ (88%)

$^{40}K \longrightarrow {}^{40}Ar$ (12%)

The $\frac{^{40}Ca}{^{40}K}$, and $\frac{^{40}Ar}{^{40}K}$ ratios should be measured

Difficulties:
- ^{40}Ca common, originates not only form the decay of ^{40}K
- ^{40}Ar is a noble gas, can escape.

Radiocarbon method (T = 5568 years)

The ^{14}C is constantly produced in the atmosphere by the cosmic radiation.

Equilibrium abundance (CO_2) in the air: $^{14}C/^{12}C = 1{,}2\cdot 10^{-12}$.

Because of the continuous metabolism this concentration is present in the living creatures.

After the death, the metabolism stops, the feed of the ^{14}C stops as well, it only decays.

The time elapsed since the death can be determined by measuring the radiocarbon concentration:

Here t is **the time since the death**,

T is the half life of ^{14}C.

$$\frac{N(^{14}C)}{N(^{12}C)} = 1{,}2\cdot 10^{-12}\left(\frac{1}{2}\right)^{\frac{t}{T}}$$

Tritium method (T = 12,26 years)

The ^{3}H is constantly produced in the atmosphere by the cosmic radiation.

Equilibrium abundance (H_2O) in the air $^{3}H/^{1}H = 1\cdot 10^{-18}$.

This concentration remains in the surface waters (rivers, lakes etc.) because of continuous exchange (rain).

The age of the **underground water** can be determined by measuring its tritium concentration.

Here t is the time since the water went underground,

T is the half life of tritium.

$$\frac{N(^{3}H)}{N(^{1}H)} = 1\cdot 10^{-18}\left(\frac{1}{2}\right)^{\frac{t}{T}}$$

Note: age of dead living creatures cannot be determined, since the H-metabolism continues after the death!

Thank you for your attention !

These slides are **uploaded** in the „Files" menu item of the Teams Group: *Nuclear and Reactor Physics Fundamentals*, in Channel: *Nuclear Physics 2. Radioactivity (15. Oct.)*

At the end of the slides there are some „*Self-test questions*". Please try to answer them to check your own understanding.

Self-test questions

1. Select the negative beta decays from the following list:
   ${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn}+\ldots$, ${}^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne}+\ldots$, ${}^{137}_{55}\mathrm{Cs} \rightarrow {}^{137}_{56}\mathrm{Ba}+\ldots$
   ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th}+\ldots$ ${}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni}^{*}+\ldots$ ${}^{60}_{28}\mathrm{Ni}^{*} \rightarrow {}^{60}_{28}\mathrm{Ni}+\ldots$
2. After the decay of ^{226}Ra, the emitted α-particle has a kinetic energy of 4,52 MeV. How much is the Q-value of the decay?
3. Can a homogenous radioactive sample emit α-particles with more than one energy? Elaborate the answer.
4. Sometimes it is said that during β^- decay a neutron decays into a proton, and during β^+ decay a proton decays into a neutron. Why is this explanation incorrect?
5. If N_1 is the number of the radioactive isotopes that decay by electron capture, and N_2 that of those, which decay by positron emission, which number would be larger? N_1 or N_2? Why?
6. The ${}^{40}_{19}\mathrm{K}$ nucleus is radioactive: with 89,3% probability it decays by β^- decay, with 10,7% it decays by electron capture. How is this possible? It can not be at both sides of the energy-valley at the same time!

Self-test questions (contd.)

7. A former unit of activity was 1 Ci (Curie), which was the activity of 1 g ^{226}Ra. Calculate this activity, and express it in Bq! (Half-life of ^{226}Ra: 1602 years)
8. An α-counter detects 40% of the emitted α-particles from a ^{226}Ra sample. During one minute it counts 400.
   a) What is the activity of the sample?
   b) What is the precision of this value (in %)
   c) What will be precision if we would measure 100 minutes?
9. Why do only 4 large decay-chains exist in nature?
10. Why are no β^+-decays and electron captures in the 4 large decay chains existing in nature?
11. During the operation of an NPP, ^{135}I and ^{135}Xe isotopes are formed. When the chain reaction stops, they form a decay chain: $^{135}\mathrm{I} \rightarrow {}^{135}\mathrm{Xe} \rightarrow {}^{135}\mathrm{Cs}$. The half-lives: 6,7 h (^{135}I), 9,2 h (^{135}Xe). Determine the time behaviour of the Xe isotope after the chain reaction stops. Will the number of Xe nuclei have a maximum? If yes, when?

Self-test questions (contd.)

12. Pierre and Marie Curie extracted 1 g ^{226}Ra from uranium ore. How much ^{238}U was in the ore? (Suppose that their extraction efficiency was 100%!) Half-lives: 4,5 billion years (^{238}U), 1602 years (^{226}Ra)
13. Now the abundance of ^{235}U in natural uranium is 0,71%. Long time ago it was higher, reached also 5%. Then, the natural uranium could form a „natural nuclear reactor" with the ground water as moderator. (Remnants of such a natural nuclear reactors have been found in Oklo /South Africa/). How long ago could that happen? Half-lives: 4,5 billion years (^{238}U), and 710 million years (^{235}U)
14. A wine-dealer argues that an old bottle of wine is 30 years old. Someone takes a small sample of the wine using a syringe through the cork, and finds that the ^{3}H concentration is $0{,}5\cdot10^{-18}$. How old is the wine?