Nuclear Reactions

Nuclear and Reactor Physics Fundamentals
(BMETE80MX00)

Prof. Csaba Sükösd
honorary professor
sukosd@reak.bme.hu

Budapest University of Technology and Economics
Institute of Nuclear Techniques (BME NTI)

Contents

Generalities, notations

Importance of nuclear reactions:
- most of our nuclear knowledge comes from studying nuclear reactions
- Practical applications use nuclear reactions to produce nuclear power, or radioactivity (for non-power applications)

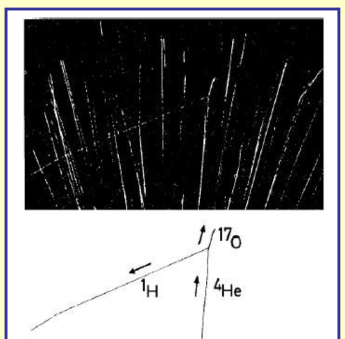
First artificial nuclear reaction:
E. Rutherford (1919)

Observed in cloud chamber

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

α-particle from radium source

Nitrogen gas in the cloud chamber

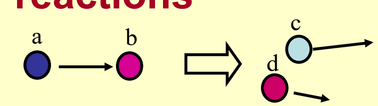


Source: Blackett and Lees

Nuclear reactions

Generally:

$a + b \longrightarrow c + d + \ldots$

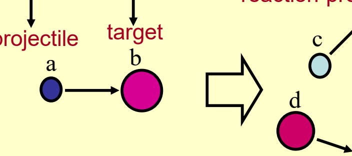


Initial state ⟶ Final state

(Interaction of more than 2 particles is not probable)

Usually one of the particles in the laboratory is at rest: the other moves

projectile target



reaction products

Initial state

Notation: b (a, c) d

target (at rest)

Final state

Scatterings: special nuclear reactions

Characteristics: a = c, (and b = d),
The type (composition) of the particles does not change

Elastic scattering: the particles (nuclei) do not get excited, total kinetic energy is conserved

Inelastic scattering: nuclei get excited (followed by γ-decay), the total kinetic energy is NOT conserved.

Examples	Description	Notation
$n+{}^{235}_{92}U \Rightarrow {}^{235}_{92}U+n'$	elastic n-scattering (n,n')	${}^{235}_{92}U(n,n')\,{}^{235}_{92}U$
$n+{}^{235}_{92}U \Rightarrow {}^{235}_{92}U+n'+\gamma$	inelastic n-scattering (n,n'γ)	${}^{235}_{92}U(n,n'\gamma)\,{}^{235}_{92}U$
$n+{}^{235}_{92}U \Rightarrow {}^{236}_{92}U+\gamma$	n-capture with γ-emission, radiating capture, (n,γ)	${}^{235}_{92}U(n,\gamma)\,{}^{236}_{92}U$
$\alpha+{}^{9}_{4}Be \Rightarrow {}^{12}_{6}C+n$	(α,n) reaction	${}^{9}_{4}Be(\alpha,n)\,{}^{12}_{6}C$
$n+{}^{59}_{27}Co \Rightarrow {}^{58}_{27}Co+2n$	(n,2n) reaction	${}^{59}_{27}Co(n,2n)\,{}^{58}_{27}Co$

Reaction energy, activation energy

The conserved physical quantities in nuclear reactions :

- **number of nucleons** (barionic charge)
- **electrical charge**
- **leptonic charge** (if electron, positron, neutrino etc. is participating in the reaction)
- **energy** (taking $E = mc^2$ into account)
- momentum
- angular momentum

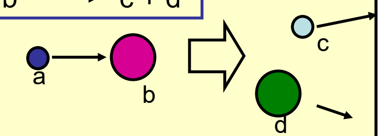
$${}^{14}_{7}N+{}^{4}_{2}He \rightarrow {}^{17}_{8}O+{}^{1}_{1}H$$

$\left.\begin{array}{l}\text{energy}\\ \text{momentum}\\ \text{angular momentum}\end{array}\right\}$ Kinematical parameters

Energy conservation

The nuclear reaction: $\boxed{a + b \longrightarrow c + d}$

M_a, M_b etc. rest masses,
T_a, T_b etc. kinetic energies



The conservation of energy :

$$(M_a \cdot c^2+T_a)+(M_b \cdot c^2+T_b) = (M_c \cdot c^2+T_c)+(M_d \cdot c^2+T_d)$$

Arrange the masses on the left side:

$$\boxed{(M_a+ M_b - M_c - M_d) \cdot c^2 = T_c + T_d - T_a - T_b = Q} \qquad (*)$$

The name of Q is: **reaction energy**

Its physical interpretation: according to the second equation

$$(T_c + T_d) - (T_a + T_b) = Q \qquad \boxed{Q = T_{final} - T_{initial}}$$

$Q > 0 \longrightarrow$ The reaction produces kinetic energy (exotherm, exoerg, „energy producing" reaction)

$Q < 0 \longrightarrow$ The reaction consumes kinetic energy (endotherm, endoerg, „energy consuming")

$Q = 0 \longrightarrow$ Kinetic energy does not change in the reaction (for example: elastic scattering)

$$\boxed{(M_a+ M_b - M_c - M_d) \cdot c^2 = T_c + T_d - (T_a + T_b) = Q} \qquad (*)$$

Energy threshold at endoterm reactions $(Q<0)$.
Since $T_c + T_d \geq 0$, we get $(T_a + T_b) \geq -Q > 0$.
In words: the particles in the initial state must have at least that much kinetic energy for the reaction to occur!
(In laboratory system usually more kinetic energy is needed, since the momentum should also be conserved)

The reaction energy and the masses of the particles:
From the (*) equation $Q = (M_a+ M_b - M_c - M_d) \cdot c^2$

$$\boxed{Q = (M_{initial} - M_{final}) \cdot c^2}$$

Thus, the reaction energy is determined by the masses!

Comment: Here M_a, M_b etc. are not always rest-masses! For example if the particle d is generated in an excited state of E_x excitation energy, then $M_d = M_d(0) + E_x/c^2$

Rest mass in ground state

Activation energy (for charged reaction partners)

One of the main reactions for the fusion energy production:

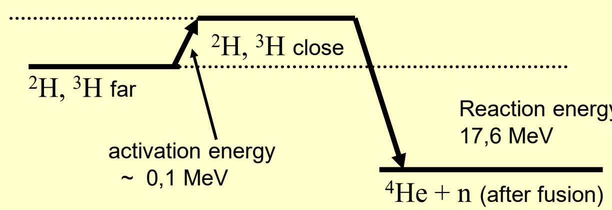
$$ {}_1^2\mathrm{H} + {}_1^3\mathrm{H} \rightarrow {}_2^4\mathrm{He} + \mathrm{n} + 17{,}6\ \mathrm{MeV} $$

The reaction is exothermic, but it does not occur easily!

Cause: the nuclear interaction is of short range, which means that the reaction partners have to get close.

Because of the Coulomb-repulsion they need (kinetic) energy

The energy plot is:



Lab. System, CM system, Kinematics

Basis: energy- and momentum-conservation

Energy-conservation (we treated it already)

Momentum-conservation: $\boxed{\mathbf{p}_a + \mathbf{p}_b = \mathbf{p}_c + \mathbf{p}_d}$ vector-equation!!

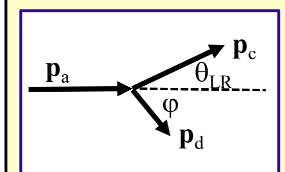
Choice of the coordinate system:

Laboratory system ⟺ Center of mass system (CM)

(Here are the measurements)

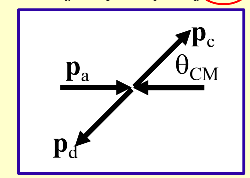
The target is **at rest** usually,

i.e. $\mathbf{p}_b = 0$, $\mathbf{p}_a = \mathbf{p}_c + \mathbf{p}_d$



(The „natural” coordinate system)

$\mathbf{p}_a + \mathbf{p}_b = \mathbf{p}_c + \mathbf{p}_d = 0$

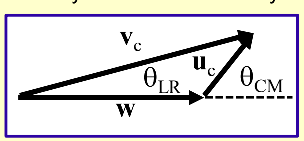


Of course $\theta_{LR} \neq \theta_{CM}$

The relative velocity of the two system is: $\mathbf{w} = \dfrac{\mathbf{p}_a + \mathbf{p}_b}{M_a + M_b}$

On the right-hand side we have the momenta in the laboratory system.

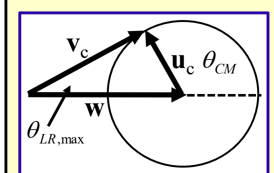
The angles of the Lab. System and the CM system are also related:



Here $\mathbf{v}_c$ is the velocity of the c particle in the laboratory system, and $\mathbf{u}_c$ is its velocity in the CM system

Obviously $u_c \sin\theta_{CM} = v_c \sin\theta_{LR}$, fromwhere $\sin\theta_{LR} = \dfrac{u_c}{v_c}\sin\theta_{CM}$

From this we get $|\sin\theta_{LR}| \leq \dfrac{u_c}{v_c}$



If $u_c < v_c$, then the scattered particles can fly only in limited angular region in the Lab.system!
(Irrespective of the value of θ_{CM})

The condition for this is: $|w| > |u_c|$

This is happening if $m_{\text{projectile}} > m_{\text{target}}$.

Example: ^{1}H(α, α')^{1}H reaction (α-particle scattering on protons)

Note on the energy threshold of endotherm reactions

We saw: $(T_a + T_b) \geq -Q$. However this is true only in CM-system, since here the total momentum is 0. Therefore T_a and T_b are energies in the CM-system.

However, in the laboratory system the total momentum is not zero, and it should be conserved also after the reaction.

Suppose that the b target is at rest, i.e. $T_b=0$.

The energy threshold of the endotherm reaction will be in the laboratory system:

$$T_a > -Q\left(1+\frac{M_a}{M_b}\right)$$

Here T_a is the kinetic energy of the projectile in the laboratory system

If $M_a >> M_b$, then endotherm reaction can occur only if $T_a >> -Q$!

Small detour:

If we use **colliding beams** of opposite momenta, then **Lab.System = CM System!**

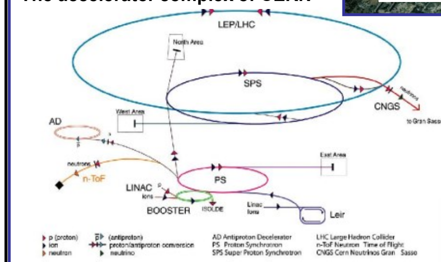
Extremely high energy-concentration can be achieved with colliding beams (used in particle physics for creation of exotic, high mass particles etc.)

CERN

(Geneva, Switzerland)

The largest accelerator-complex of the world uses also colliding beams

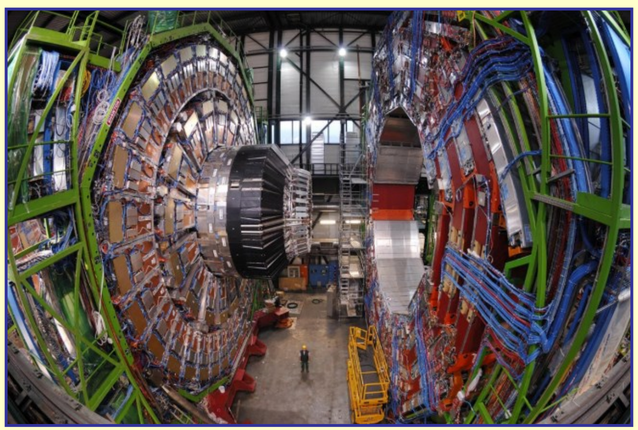
The accelerator complex of CERN



LHC:

Large Hadron Collider
In a tunnel ~100 m deep underground
Circumference: ~ 27 km.
Protons collide
Total energy: $14\cdot10^{12}$ eV

One of the 4 huge detectors of the LHC at CERN:
the **Compact Muon Solenoid** (CMS) - just to amaze you… ☺



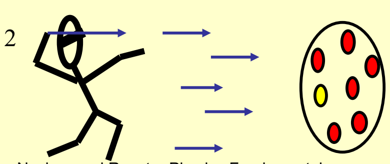
Cross section concept

Nuclear reactions are stochastic processes.
(Remember: radioactive decay is also a stochastic process!)
They can be described only by random variables.

Introductory model:
Consider a „darts" panel with A = 1 m² area. There are N = 100 small-area (σ = 1 cm²) objects scattered on that, randomly. A blindfold player throws arrows at the panel. From the many arrows thrown, 200 arrows hit the panel during an hour (n = 200/h). Estimate, how many objects are expected to be hit in that hour?

$$R=\frac{200}{10^4\ \mathrm{cm}^2}\cdot 100\cdot 1\ \mathrm{cm}^2=2$$

$$R=\frac{n}{A}\cdot N\cdot\sigma$$



Denote the n/A quantity by ϕ. $R = \frac{n}{A} \cdot N \cdot \sigma$

Interpretation of ϕ: number of particles flying through unit area during unit time.

The name of ϕ is: scalar **flux** (of particles)

The unit of the flux is: 1/(time·area) = [1/(s·cm²)]

The number of the expected „hits" in unit time: $\boxed{R = \phi \cdot N \cdot \sigma}$

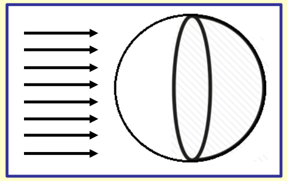
The name of R is: **reaction rate**

Its unit is: 1/time e.g. [1/s]

The name of σ is :

total **microscopic cross section**

Its unit is: area e.g. [cm²]



Order of magnitude of the cross section of the nuclei: ~ R^2, (R ~ 10^{-14} m)

$R^2 = \left(10^{-14}\ \text{m}\right)^2 = 10^{-28}\ \text{m}^2 = 10^{-24}\ \text{cm}^2$ $\boxed{1\ \text{b} = 10^{-24}\ \text{cm}^2}$ (1 barn)

General **definition** of the microscopic cross-section: $\boxed{\sigma = \frac{R}{N \cdot \phi}}$

Very important:

Although it was introduced using a simple model, it is **NOT** the geometrical surface area of the nucleus!!!

Even not the total geometrical surface area of the target + projectile system!

Every particular nuclear-reaction has its own cross-section!

(since R is determined by the actual reaction rate)

For example the following two reactions have completely different cross-sections, although the target and the projectile are the same!

n + ^{235}U $\longrightarrow$ ^{236}U + γ (radiating capture)

n + ^{235}U $\longrightarrow$ ^{235}U + n'+ γ (inelastic n-scattering)

Moreover, the cross-section usually depends also on the energy of the projectile! $\sigma(E)$

The microscopic cross-section (σ) is a measure of the probability for a particular reaction to occur. Unit is [cm²], or barn [10^{-24} cm²]

Macroscopic cross-section

Definition $\boxed{\Sigma = \rho \cdot \sigma}$ $\left(\frac{1}{\text{cm}^3} \cdot \text{cm}^2\right)$. Here ρ is the **particle-density**

The name of Σ is: total **macroscopic cross-section**.

The unit of the macroscopic cross-section is: 1/length (!!!)

Consider a parallel beam of particles with a ϕ_0 initial flux!

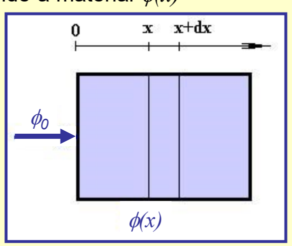
We calculate the flux at a distance x inside a material $\phi(x)$

Consider first an A area of the sample

The number of target nuclei in the dx layer is: $dN = \rho \cdot A \cdot dx$

The reaction rate in the dx layer:

$$dR = \phi(x) \cdot dN \cdot \sigma = \phi(x) \cdot \rho \cdot \sigma \cdot A \cdot dx$$



The number of particles reacted in the $(x,x+dx)$ layer will be missing from the incoming flux:

$$-d\phi = \frac{dR}{A} = \phi(x) \cdot \rho \cdot \sigma \cdot dx = \phi(x) \cdot \Sigma \cdot dx$$

This leads to: $\frac{d\phi}{dx} = -\Sigma \cdot \phi(x)$

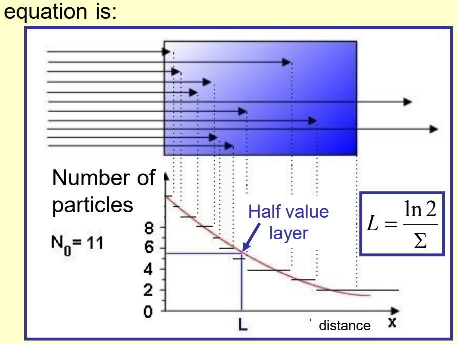
The solution of this diff. equation is:

$$\boxed{\phi(x) = \phi_0 e^{-\Sigma \cdot x}}$$

This is the **exponential attenuation law**

Σ is also called :

Linear attenuation coefficient



Note also:

The total macroscopic cross-section **can be measured** by measuring the attenuation of the flux!

Double additivity of the cross-sections

Every particular nuclear reaction has its own cross-section!

I. Additivity: same reaction partners, different reactions, (mutually exclusive)

Example: the reactions of a neutron (with given energy) and a target nucleus can be grouped in 2 mutually exclusive groups:
- Scattering (s) σ_s - the n is still there after the reaction
- Absorption (a) σ_a - the n „disappears" in the reaction

After the absorption several „outcomes" are also possible. For example:
- radiating capture (c) σ_c (n,γ) reaction
- fission (f) σ_f (n-induced fission)
- etc.

The definition of the microscopic cross section: $\boxed{\sigma = \frac{R}{N \cdot \phi}}$

If the reaction partners are the same, N and ϕ are also the same.

Thus $R_{all} = R_{scattering} + R_{absorption}$ (**all** possibilities are listed)

Therefore the total cross section (σ_t):

$$\sigma_t = \frac{R_{all}}{N \cdot \phi} = \frac{R_{scattering} + R_{absorption}}{N \cdot \phi} = \frac{R_{scattering}}{N \cdot \phi} + \frac{R_{absorption}}{N \cdot \phi} = \sigma_s + \sigma_a$$

Summarised: $\sigma_t = \sigma_s + \sigma_a$

Similarly: after the absorption (in our example) there are two possible reactions, therefore $R_{absorption} = R_c + R_f$

Thus we get $\sigma_a = \sigma_c + \sigma_f$

Substituting this above: $\sigma_t = \sigma_s + \sigma_c + \sigma_f$

I. additivity: $\sigma_t = \sigma_1 + \sigma_2 + \sigma_3 + \ldots + \sigma_n$

Here σ_i are the cross sections of **all mutually exclusive** reactions, and σ_t is the total cross section.

Similar additivity holds for the **macroscopic** cross sections too.

II. Additivity:

This relates to a target **material** composed of **different elements!**

Suppose, that a n-flux interacts with a **composite target**, which contains several, different materials.

The particle densities are: $\rho_1, \rho_2, \rho_3, \ldots \rho_N$

The different materials attenuate the n-flux independently from each-other, which means

$$\phi(x) = \phi(0) \cdot e^{-\Sigma_t(1) \cdot x} \cdot e^{-\Sigma_t(2) \cdot x} \cdot e^{-\Sigma_t(3) \cdot x} \ldots$$

From this we get easily:

$$\boxed{\Sigma_t(all) = \Sigma_t(1) + \Sigma_t(2) + \Sigma_t(3) + \ldots + \Sigma_t(N)}$$

This is the second additivity of the cross sections – related to composite materials

Warning! Such kind of additivity can **NOT** be established for the **microscopic cross sections**!

Mean free path and Σ_{total}

Mean free path: the average distance which can be done by a particle without any interaction

We saw earlier: $\phi(x) = \phi_0 e^{-\Sigma_t \cdot x}$

This shows what part of the initial flux reaches the x distance without any interaction.

The „probability density" for one single particle reaching x distance without any interaction: (Note: this probability density is not „normalised")

$$\frac{\phi(x)}{\phi_0} = e^{-\Sigma_t \cdot x}$$

The **expectation value** of the distance without interaction:

$$< x > = \frac{\int_0^\infty x \cdot e^{-\Sigma_t x} dx}{\int_0^\infty e^{-\Sigma_t x} dx} = \frac{\left(\frac{1}{\Sigma_t^2}\right)}{\left(\frac{1}{\Sigma_t}\right)} = \frac{1}{\Sigma_t}$$

The mean free path: $\boxed{\Lambda = \frac{1}{\Sigma_t}}$

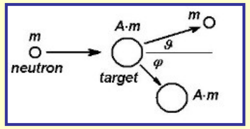
Neutron-induced special reactions

Elastic scattering

Momentum and kinetic energy are conserved!

Kinetic energy conservation:

$$\frac{p_1^2}{2m}=\frac{p_2^2}{2m}+\frac{P^2}{2A\cdot m}$$



General case:

Momentum conservation

$p_1 = p_2\cdot\cos\vartheta + P\cdot\cos\varphi$ (x-direction)

$0 = p_2\cdot\sin\vartheta - P\cdot\sin\varphi$ (y-direction)

Special case:

The largest energy transfer occurs when $\vartheta=\pi$ and $\varphi=0$

Momentum conservation: $p_1 = P - p_2$ („head-on" collision)

Substituting it in the energy-equation we get finally the solution : $\frac{p_2}{p_1}=\left(\frac{A-1}{A+1}\right)$

This yields for the final energy after a „head-on" collision: $\boxed{\frac{E_2}{E_1}=\left(\frac{A-1}{A+1}\right)^2}$

If $A = 1$ (the target is a proton), $E_2 = 0$.
This means that if the neutron collides „head-on" with a proton, it can loose its whole energy in one single collision.

This equation shows also, that if $A >> 1$ then $E_2 \sim E_1$.
With other words: heavy nuclei cannot slow down neutrons, only light nuclei.

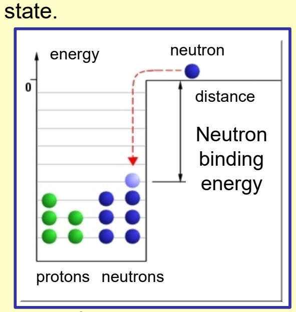
The theory of slowing down the neutrons is essential for the operation of nuclear reactors.

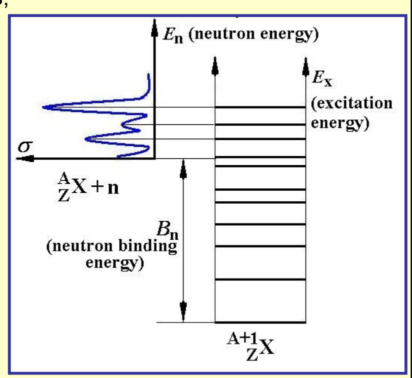
Of course it is much more complex than the simple „head-on" special case presented here.

Neutron-induced special reactions (contd.)

Neutron absorption

Neutrons are bound in the nucleus.
If an additional neutron enters, its binding energy leaves the new nucleus in an **excited** state.

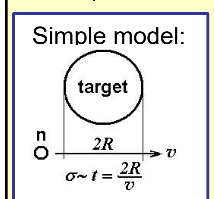




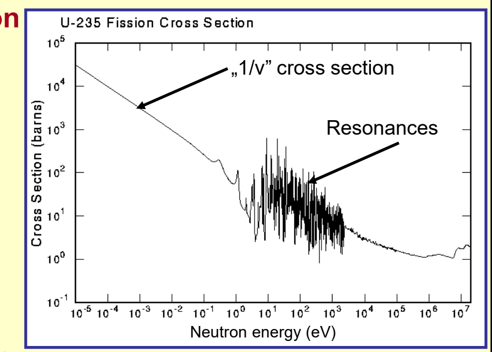
Special cross-sections

(1/v) cross section

Example:



$$\sigma(E)\sim\frac{1}{v}=\frac{1}{\sqrt{2mE}}$$



then $\ln\sigma(E)\sim\mathrm{const}-\frac{1}{2}\ln E$ This is **linear** in a $\ln\sigma - \ln E$ plot

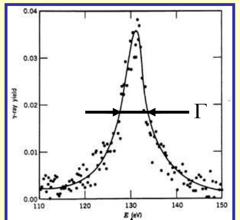
Reactions with **no threshold and no activation energy** have this feature in the low energy region

Resonances

Cause: an **existing (excited) state** is formed in the composite nucleus

Shape of the distinct resonances:
(Breit-Wigner formula)

$$\boxed{\sigma(E)=\sigma(E_0)\cdot\frac{\left(\Gamma/2\right)^2}{\underbrace{(E-E_0)^2}_{\text{„position”}}+\underbrace{\left(\Gamma/2\right)^2}_{\text{„width”}}}}$$



E_0

The „width” of the resonance and the τ life-time of the excited state are related: $\Gamma\cdot\tau\approx\frac{h}{2\pi}$

or with the T half life: $\boxed{\Gamma\cdot T\approx\frac{h}{2\pi}\cdot\ln 2}$

Here h is the Planck-constant

The reason for this relationship:
the **Heisenberg uncertainty principle** (quantum mechanics)

Thank you for your attention !

These slides are **uploaded** in the „Files” menu item of the Teams Group: *Nuclear and Reactor Physics Fundamentals*,
in Channel: *Nuclear Physics 3. Nuclear Reactions (22. Oct.)*

At the end of the slides there are some „*Self-test questions*”. Please try to answer them to check your own understanding.

Self-test questions

1. Is there any difference between the following reactions? Give rationale of your answer!
   $p(\alpha,\alpha')p$, $^{1}H(^{4}He,^{4}He')^{1}H$, $\alpha(^{1}H,^{1}H')\alpha$
2. Complete the following reactions!
   $^{59}Co(n,\gamma)xx$, $^{16}O(^{2}H,^{6}Li)xx$, $^{58}Ni(n,xx)^{57}Ni$, $^{58}Ni(xx,yy)^{61}Zn$
3. What kind of reaction occurs in the Rutherford experiment?
4. How can energy be produced from nuclear reactions, if the energy is conserved?
5. The main reaction for fusion energy production is:
   $$^{2}_{1}H+^{3}_{1}H\rightarrow^{4}_{2}He+n+17{,}6\,\text{MeV}$$
   How is the released energy shared between the reaction products in Center of Mass System?
6. What kind of particles are accelerated by the LHC? To what energy? For what purpose?

Self-test questions (cont.)

7. 36% of the initial flux of thermal neutrons (n_{th}) is absorbed in 20 cm thick water layer.
   a) What part of the neutron-flux is absorbed in a 5 m thick water-layer (like in the Training Reactor of BME)?
   b) Calculate the macroscopic absorption cross-section of water for n_{th}.
   c) Neglect the absorption by the oxygen, and calculate the microscopic absorption cross-section of hydrogen for n_{th}
8. Why does the total cross-section determine the mean free path? Why not the absorption cross-section for example?
9. Why is the following equation wrong?
   $\sigma_{total} = \sigma_s + \sigma_e + \sigma_i + \sigma_a + \sigma_c$
   Here the letters in the indices denote the following:
   s – neutron scattering
   e – neutron elastic scattering
   i – neutron inelastic scattering
   a – neutron absorption
   c – neutron capture: (n,γ) reaction

Self-test questions (cont.)

11. Calculate the macroscopic absorption cross section of $BaTiO_3$ for thermal neutrons! Data: σ_a(Ba) =1,3 barn, σ_a(Ti) = 6,09 barn, σ_a(O) = 0,19 millibarn, molar mass: 233,192 g/mol, density = 6,02 g/cm^3.
12. Suppose that a 1 MeV neutron slows down in graphite, (^{12}C) with only „head-on" collisions.
    How many collisions are needed to reach 0,1 eV energy?
    How many collisions, if it would slow down in heavy water?
13. What are the conditions that 1/v cross-section occurs in a reaction?
14. What is the cause of the resonances in the cross sections at certain energies?
15. What are the characteristic parameters of the resonances?