



# Basic Properties of Nuclei

## Nuclear and Reactor Physics Fundamentals

(BMETE80MX00)

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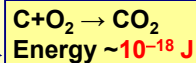
## Contents

- A short **historical** overview
- The **size** of the nucleus
- The **composition** of the nucleus
- The **mass** of the nucleus
- **Binding energy** of the nucleus
- **Angular momentum and parity**
- **Magnetic dipole momentum**
- **Excited states**, energy levels

## At the end of the 19<sup>th</sup> century the beautiful „castle” of the science was ready

**Chemistry:** atom- molecule hypothesis,  
Avogadro-number, mole  
**Constancy of the chemical elements**

**Physics:** Thermodynamics, (industrial revolution)  
**Energy conservation** (impossibility of perpetual mobile)



Size of the (hypothetical) atoms:  
E.g. gold density  $\sim 19700 \text{ kg/m}^3$   
molar mass: 0,197 kg  
 $0,197 \text{ kg} \rightarrow 6 \cdot 10^{23} \text{ atoms}, 10^{-5} \text{ m}^3$

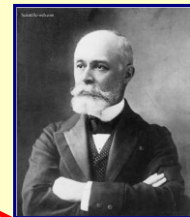
Volume of one single atom:  
 $V = (10^{-5} \text{ m}^3) / (6 \cdot 10^{23}) = 16 \cdot 10^{-30} \text{ m}^3$   
 $2R = \sqrt[3]{16 \cdot 10^{-30}} \approx 2,52 \cdot 10^{-10} \text{ m}$

**Atomic radius  $\sim 10^{-10} \text{ m}$**



## 1896: Henri Becquerel Discovery of the Radioactivity

The uranium constantly radiates energy without any outer energy source (**about million times more** than burning a C atom!) **Energy conservation?**



When uranium decays, **other elements** (helium and thorium) are being created  
**Constancy of the elements?**

**Two strong bastions were destroyed the same time by the misterious radioactivity!**



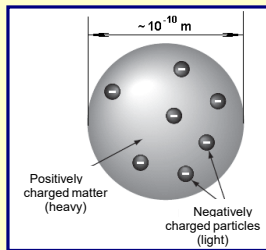
## 1897: J. J. Thomson Discovery of the electron

- There are electrons in **EVERY** material  
**The atoms are not indivisible!**
- Measurements of  $q/m$

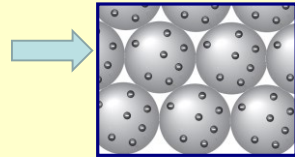


J.J. Thomson (1856-1940)

### Thomson-model of the atom



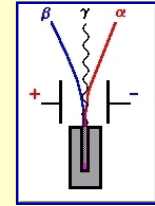
Electric charge mass  
 $q/m = -17589,20 \cdot 10^{-15} \text{ C/kg}$  for **electrons**  
 $q/m = + 9,58 \cdot 10^{-15} \text{ C/kg}$  for **H<sup>+</sup> ion**  
 (for other positive ions still smaller!)  
 But!! Atoms are electrically neutral,  
 therefore  $|q|_{\text{electron}} = |q|_{\text{ion}} \rightarrow m_{\text{electron}} \ll m_{\text{ion}}$



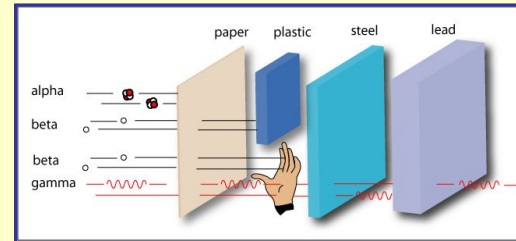
## Radiation components

Three types of particles are emitted:

- $\alpha$  - particles: **He<sup>++</sup>** ions
- $\beta$  - particles: high energy **electrons**
- $\gamma$  - radiation: **electromagnetic** (photons)



The penetration distance (range) are different:



## The size of the nucleus

### 1911: Rutherford experiment

$\alpha$ - particles scattered on thin gold (Au) foil

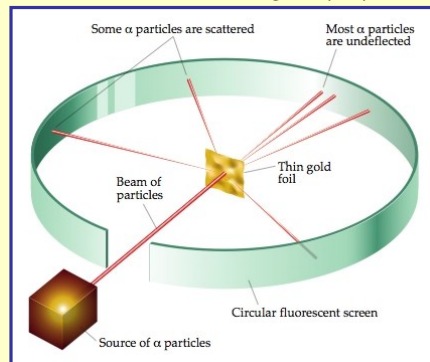


E. Rutherford (1871-1937)

The experimental apparatus should be in vacuum

### Why gold?

VERY thin foils can be prepared from gold!  
 (few atomic layer only)



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### What was expected?

The Coulomb interaction between the positively charged part of the atom and the alpha-particle: a „Coulomb-hill” to climb.

The maximal height of the „Coulomb hill”:

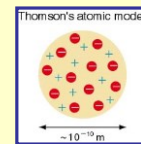
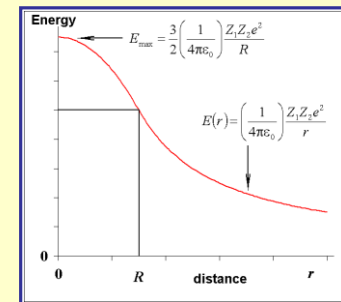
$$E_{\max} = \frac{3}{2} \left( \frac{1}{4\pi\epsilon_0} \right) \frac{Z_1 Z_2 e^2}{R}$$

$Z_1 = 2$  (atomic number of He)  
 $Z_2 = 79$  (atomic number of Au)

$$\left( \frac{1}{4\pi\epsilon_0} \right) = 9 \cdot 10^9 \text{ J}\cdot\text{m}/\text{C}^2$$

$e = 1,6 \cdot 10^{-19} \text{ C}$  (Coulomb)

„Thomson's-model” of the atom (pudding model):  
 the radius of the positively charged part is approx.  
 the radius of the atom  $\sim 10^{-10} \text{ m}$  (order of magnitude)



If  $R = R_{\text{atom}} \sim 10^{-10} \text{ m}$   
 then  $E_{\text{max}} \sim 5,5 \cdot 10^{-16} \text{ J}$

But:  $E_{\text{alpha}} \sim 7700,0 \cdot 10^{-16} \text{ J}$

Like cannon ball  
 through a paper!



#### Experimental result:

There were also „back-scattered” particles!

#### Conclusion:

The potential hill should be „higher”, than the energy of the  $\alpha$ -particle!

$$\text{That is: } \frac{3}{2} \left( \frac{1}{4\pi\epsilon_0} \right) \frac{Z_1 Z_2 e^2}{R} > E_{\text{alpha}}$$

From this we have:

$$R < \frac{3}{2} \left( \frac{1}{4\pi\epsilon_0} \right) \frac{Z_1 Z_2 e^2}{E_{\text{alpha}}}$$

Using  $E_{\text{alpha}} \sim 7700 \cdot 10^{-16} \text{ J}$  we calculate:  $R < 10^{-14} \text{ m}$ , which is about **ten-thousand** times smaller than the size of the atoms!

**The mass and the positive electric charge is concentrated in the very small atomic nucleus!**

#### Hofstdter experiment (1950-54) :

Scattering of high-energy electrons on different materials  
 Even the charge distribution could be determined

Why was high-energy ( $\sim 300 \text{ MeV}$ ) electrons needed?

Electrons are even not repelled by the nucleus!!!

Remember: the resolving power of a microscope depends on the wave-length ( $\lambda$ ) used!

The wave-length of a particle (de Broglie):

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Resolving (spatially) small details, small wave-length is needed: **large velocity**, i.e. large energy.

Rutherford was lucky: because of the much **larger mass** of the alpha-particle, its wave-length was small enough to discover the atomic nucleus!

#### Results of the Hofstdter experiment:

- The **central density ~ constant**

•  **$R = r_0 \cdot A^{1/3}$** , where

- $A$  = number of nucleons (mass-number)

•  $r_0 = 1,07 \cdot 10^{-15} \text{ m} = 1,07 \text{ fm}$ .

**Note:**  $V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} r_0^3 A$

this means:  **$A \sim V$**  (volume)

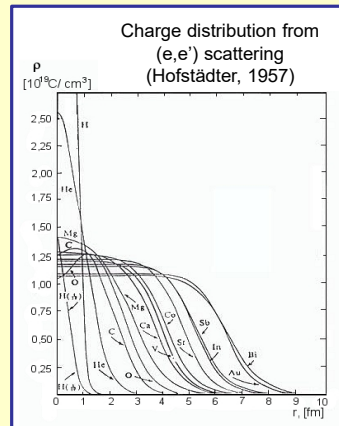
The charge-density can be well described by a

Fermi-function: 
$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{d}}}$$

where

$R$  is the nuclear radius, ( $\sim r_0 \cdot A^{1/3}$ )

$d$  is the surface „diffuseness” (~constant)



## The composition of the nucleus

$Z$  protons, (‘atomic number’)

$N$  neutrons

} **nucleons**

$A = Z + N$  (mass-number, number of nucleons)

Notation:  $\begin{smallmatrix} A \\ Z \end{smallmatrix} X_N$  e.g.  ${}^4_2\text{He}_2$   ${}^{40}_{19}\text{K}_{21}$   ${}^{238}_{92}\text{U}_{146}$

Redundant,  ${}^{238}\text{U}$  alone is sufficient !

	proton	neutron
mass	$1,67265 \cdot 10^{-27} \text{ kg}$	$1,67495 \cdot 10^{-27} \text{ kg}$
charge	$+e$	0

#### Notations:

- nuclei with the same number of protons ( $Z$ ) : **ISOTOPES**
- nuclei with the same number of nucleons ( $A$ ) : **ISOBARS**
- nuclei with the same number of neutrons ( $N$ ): **ISOTONES** (rarely used)

## The mass of the nucleus

The „mass-defect”:  $M(Z, A) = Z \cdot m_{\text{proton}} + (A - Z) \cdot m_{\text{neutron}} - \Delta M$

**Cause:** the nucleus is a bound system, energy is needed to take it apart.

Einstein's relation:  $E = m \cdot c^2$ . If energy is needed, then also mass is needed to take the bound nucleons apart!

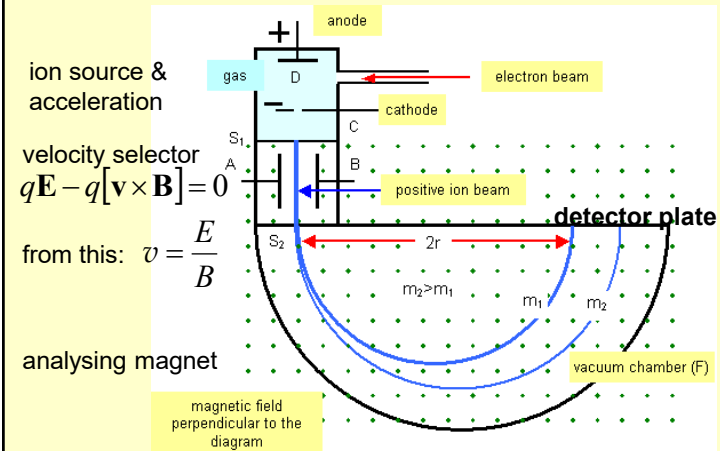
With measuring the mass-defect (precise measurement of nuclear mass) the **binding energy** of the nucleus can be determined:  $B = \Delta M \cdot c^2$

### Measuring the mass of the atoms:

With mass-spectrometers (mass-spectroscopes)

- The atoms get first ionised,
- The ions get accelerated with electrical fields
- The accelerated ions will be deflected by electric and magnetic fields
- The mass can be determined from the deflection of ions

## Aston's mass spectrometer (1919)



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The principle of the analyser:

$$\frac{mv^2}{r} = q \cdot v \cdot B$$

centripetal force = Lorentz force

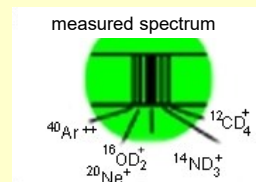
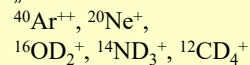
From this we get:  $r = \left( \frac{m}{q} \right) \cdot \frac{v}{B}$

The spectrometers select according to  $(m/q)$

Small mass differences can be measured very precisely!

For example:  $(m/q) = 20$

„mixed” beam:



The nuclear mass is often expressed in **atomic mass unit**:

$$1 \text{ u} = \frac{M(^{12}_6\text{C})}{12}$$

The mass  $M(^{12}\text{C})$  here is the mass of the C **atom**!!!  
 (mass of 6 electrons included)

$$1 \text{ u} = 1,66043(2) \cdot 10^{-27} \text{ kg} = 931,478 \text{ MeV}/c^2$$

### Why was the $^{12}\text{C}$ atom chosen?

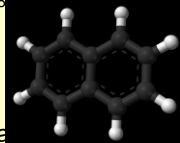
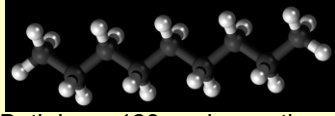
Earlier the mass of the hydrogen atom (the lightest atom) was chosen as a mass unit, then the  $M(^{16}\text{O})/16$ , and finally the conclusion was the  $M(^{12}\text{C})/12$ . Why?

Because carbon atoms can form a huge variety of molecules with different atoms! (See the whole organic chemistry)

This feature enables the determination of the mass of every atom (in this unit) **very precisely**!

### Mass-doublet method

For example:  $C_9H_{20}$  (nonane) and  $C_{10}H_8$  (naphthalene)



Both have 128 nucleons, therefore their masses are *approximately* 128 u. However, precise masses are different!

$$\text{Precisely: } M(\text{nonane}) = 9 \cdot M(C) + 20 \cdot M(H)$$

$$M(\text{naphthalene}) = 10 \cdot M(C) + 8 \cdot M(H)$$

$$\text{After subtraction: } \Delta M = -M(C) + 12 \cdot M(H)$$

$$\text{From this: } M(^1H) = \frac{M(^{12}C)}{12} + \frac{\Delta M}{12} = 1,0000... + \frac{\Delta M}{12}$$

A good measurement gives:  $\Delta M = 0,09390032 \pm 0,00000012$  u

$$\text{From this: } M(^1H) = \frac{M(^{12}C)}{12} + \frac{\Delta M}{12} = 1,00782503 \pm 0,00000001 \text{ u}$$

Very precise!! (precision is  $\sim 10^{-8}$ )

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### Binding energy of the nucleus

The **binding energy**:  $B = \Delta M \cdot c^2$  (Einstein)

By measuring the mass-defect (mass-spectrometers) the binding energy can be determined

#### Energy and binding energy:

Einstein:  $E = m \cdot c^2$ . Since  $m \geq 0$ , the total **energy** is  $E \geq 0$ .

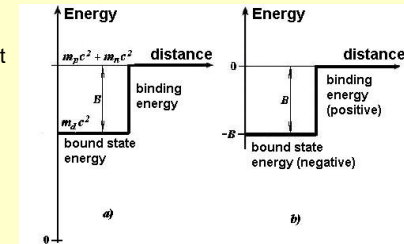
Example: look at the mass of deuteron ( $^2H$ ) and its energy!

$$m_d = m_p + m_n - \Delta M \quad (\text{multiply by } c^2)$$

$$m_d c^2 = m_p c^2 + m_n c^2 - B$$

Usually the zero point of the energy axis will be chosen at the unbound system (right side of the picture).

If so, the energy of the bound system will be **NEGATIVE**:  $E = -B$



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### Binding energy of nuclei

(Semi-empirical binding energy formula of Weizsäcker)

Starting point: **nuclear density ~ constant**, thus the nucleus is like an (electrically charged) liquid drop (Liquid drop model)

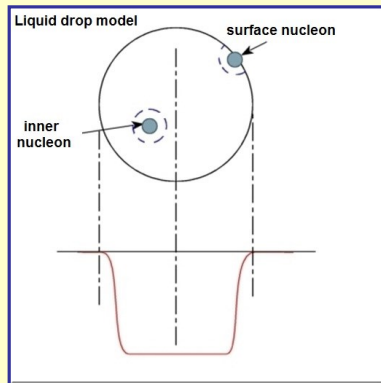
The nucleons interact only with neighbors. If all nucleon was „inner“ one, then the total binding energy would be  $B = b_V \cdot A$ .

( $b_V$  is the binding energy of one „inner“ nucleon)

The „**surface**“ nucleons are bound weaker, thus

$$B = b_V \cdot A - \beta \cdot 4\pi R^2$$

Here  $\beta$  is a constant.



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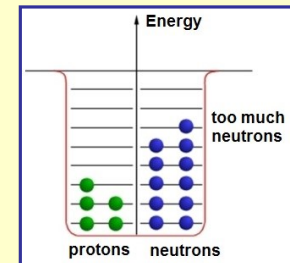
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So far only the nuclear interaction was taken into account.

The nucleus has also  $Ze$  electric charge, and it makes the binding weaker (because of the **Coulomb-energy** due to the mutual repel of the protons):

$$B = b_V \cdot A - \beta \cdot 4\pi R^2 - \frac{3}{5} k \frac{Z^2 e^2}{R}$$

Because of quantum mechanics the Pauli principle is valid for the protons and the neutrons (at most 2 particles can be at an energy level). Too much neutron or proton (**asymmetry**) weakens the binding:



$$B = b_V \cdot A - \beta \cdot 4\pi R^2 - \frac{3}{5} k \frac{Z^2 e^2}{R} - b_A \frac{(Z - N)^2}{A}$$

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Finally: empirical fact is that nuclei are stronger bound, if their number of protons or neutrons (or both) are even (pairing energy).

$$B = b_V A - b_F \cdot 4\pi R^2 - \frac{3}{5} k \frac{Z^2 e^2}{R} - b_A \frac{(Z - N)^2}{A} + b_P \delta \cdot A^{-3/4}$$

Here  $\delta = 1$ , if the nucleus is even-even  
 $\delta = 0$ , if the nucleus is even-odd  
 $\delta = -1$ , if the nucleus is odd-odd

Use now the relation  $R = r_0 A^{1/3}$ , and unify the different constants to one constant at every member:

$$B = b_V A - b_F \cdot A^{2/3} - b_C \cdot \frac{Z^2}{A^{1/3}} - b_A \cdot \frac{(N - Z)^2}{A} + b_P \cdot \delta \cdot A^{-3/4}$$

**This is the semi-empirical binding energy formula of Weizsäcker**

The name of the different members in the formula (the value of the constants are in brackets)

- **Volume** energy ( $b_V = 2,52 \cdot 10^{-12}$  J)
- **Surface** energy ( $b_F = 2,85 \cdot 10^{-12}$  J)
- **Coulomb** energy ( $b_C = 0,11 \cdot 10^{-12}$  J)
- **Asymmetry** energy ( $b_A = 3,80 \cdot 10^{-12}$  J)
- **Pairing** energy ( $b_P = 1,49 \cdot 10^{-12}$  J)

These constants were determined empirically. With these 5 constants the binding energy of the more than 2000 nuclei (discovered so far) can be well described with a precision of 1-2 %

**Average binding energy** of one single nucleon:  $b = B/A$ .

How „deeply“ sits a nucleon in the attractive potential of the nucleus? How much is the **average energy** of one single nucleon in the nucleus?

$$\varepsilon = -b = -B/A.$$

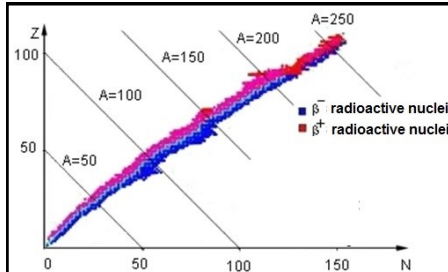
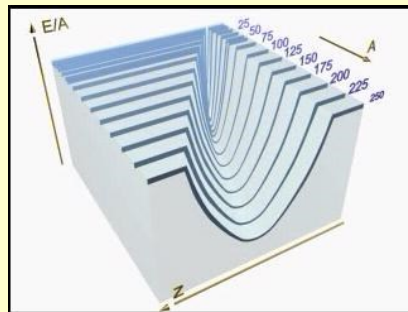
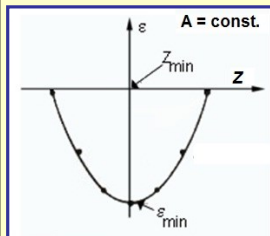
Importance: **during spontaneous processes  $\varepsilon$  decreases!**  
 (energy minimum principle, 2<sup>nd</sup> law of thermodynamics)

Since  $B$  is a function of the nuclear composition ( $Z, A$ ), thus  $\varepsilon$  is a function of those as well.  
 $\varepsilon = \varepsilon(Z, A)$ . This can be drawn as a „surface“.

$$\varepsilon = \frac{E}{A} = -\frac{B}{A} = -\frac{1}{A} \left( b_V A - b_F \cdot A^{2/3} - b_C \cdot \frac{Z^2}{A^{1/3}} - b_A \cdot \frac{(N - Z)^2}{A} + b_P \cdot \delta \cdot A^{-3/4} \right)$$

Note, that the  $A = \text{const.}$  cuts are paraboles!

$$\varepsilon(Z)_{A=\text{const.}} = a \cdot (Z - Z_{\min})^2 + \varepsilon_{\min}$$



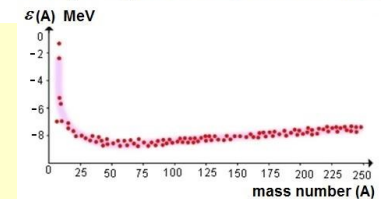
Position of  $Z_{\min}$  on the  $(N, Z)$  „map“

This helps to understand the radioactive decays!

$\varepsilon_{\min}$  in function of the mass-number ( $A$ )

This helps to understand the energy production from the nuclei (nuclear energy production)

Average energy of a nucleon in function of mass number

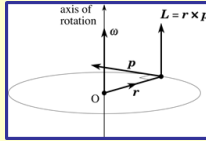




## Angular momentum and parity

In classical physics for a point-like mass

Angular momentum vector  $\vec{L} = \vec{r} \times \vec{p}$   
 Position vector  $\vec{r}$   
 Momentum vector  $\vec{p}$

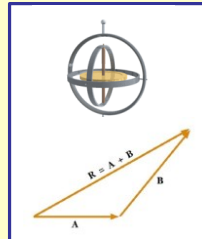


Since  $\vec{p} = m \cdot \vec{v}$ , therefore  $\vec{L} = m \cdot (\vec{r} \times \vec{v})$

But  $\vec{v} = \vec{r} \times \vec{\omega}$ , therefore  $\vec{L} = m \cdot (\vec{r} \times (\vec{r} \times \vec{\omega})) = (m \cdot r^2) \cdot \vec{\omega} = \theta \cdot \vec{\omega}$

Angular velocity vector  $\vec{\omega}$

Inertial momentum



**Importance:** the angular momentum is **constant**, if there is no external torque! (For example: gyroscope)

**Vector addition:** „triangular inequality”

$$\vec{R} = \vec{A} + \vec{B} \quad \Rightarrow \quad |A - B| \leq R \leq A + B$$

where  $A = |\vec{A}|$ ,  $B = |\vec{B}|$  és  $R = |\vec{R}|$

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## In Quantum Physics the angular momentum is quantised!

First try: Niels Bohr:  $|\vec{L}| = n \cdot \frac{h}{2\pi} = n \cdot \hbar$ , where  $n = 0, 1, 2, 3, \dots$

and  $h$  is the Planck-constant:  $h = 6,62607004 \cdot 10^{-34} \left[ \frac{\text{m}^2 \text{kg}}{\text{s}} \right]$

Well, but the angular momentum is a vector! It has not only length but also direction! If it has direction, then it has projections on the coordinate axis!

After the quantum mechanics was fully developed:

$$|\vec{L}|^2 = \ell \cdot (\ell + 1) \cdot \hbar^2 \quad \text{and} \quad L_z = m \cdot \hbar \quad \text{where} \quad -\ell \leq m \leq \ell$$

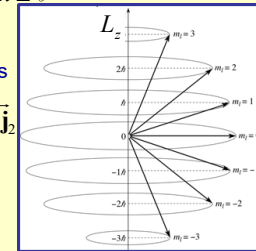
orbital quantum number  $\ell$   
 magnetic quantum number  $m$   
 possibilities  $2\ell + 1$

Addition of angular momentum vectors:  $\vec{J} = \vec{J}_1 + \vec{J}_2$

This means for the quantum numbers:

„lengths”  $|j_1 - j_2| \leq j \leq j_1 + j_2$

„projections”  $m_j = m_{j1} + m_{j2}$



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We have two „types” of angular momentum in microphysics ( $\hbar$  units):

- „orbital” momentum  $\ell = 0, 1, 2, 3, \dots$  („revolving” particles have it)
- „spin” (intrinsic angular momentum):  $s = \frac{1}{2}$  (even a free particle has it)

The spin of the neutron and the proton:

$$|\vec{s}|^2 = \frac{1}{2} \cdot \left( \frac{1}{2} + 1 \right) \cdot \hbar^2 \quad \text{and} \quad s_z = \hbar \cdot \begin{cases} +\frac{1}{2} & \text{spin „up”} \\ -\frac{1}{2} & \text{spin „down”} \end{cases}$$

## Angular momentum of nuclei

The „total” angular momentum of a nucleon (p,n) in the nucleus:  $\vec{J} = \vec{L} + \vec{s}$

According to the addition rule:  $j = l + \frac{1}{2}$ , or  $j = l - \frac{1}{2}$   
 and the projections:  $m_j = m_l + m_s$

There are  $A$  nucleon in a nucleus:  $\vec{J} = \sum_{i=1}^A \vec{j}_i = \sum_{i=1}^A (\vec{L}_i + \vec{s}_i)$

This can be very complicated but two statements are true for sure:

- If  $A$  is odd, then  $j$  is „half-integer” ( $1/2, 3/2, 5/2$  etc.)
- If  $A$  is even, then  $j$  is integer ( $0, 1, 2, 3, \dots$ )

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## BUT! Nature (fortunately) simplifies things for us!

- If number of protons are even, then for the proton-pairs:  $\vec{j}_1 + \vec{j}_2 = 0$
- If number of neutrons are even, then for the neutron-pairs:  $\vec{j}_1 + \vec{j}_2 = 0$

**Cause:** pairing interaction (p-p and n-n pairs are stronger bound)

**Consequences:**

- For even-even nuclei:  $\vec{J} = 0$  (because all nucleons are paired)
- For even-odd nuclei:  $\vec{J} = \vec{j}_{\text{odd}}$  (from the single unpaired nucleon)
- For odd-odd nuclei:  $\vec{J} = \vec{j}_1 + \vec{j}_2$  (from the two unpaired nucleon)

**Systematics** of stable nuclei ( $Z$  = proton number,  $N$  = neutron number)

	Even $N$	Odd $N$	Total
Even $Z$	156	48	204
Odd $Z$	50	4	55
Total	206	53	259

The 4 stable odd-odd nuclei are:  ${}^2_1\text{H}$ ,  ${}^6_3\text{Li}$ ,  ${}^{10}_5\text{B}$ ,  ${}^{14}_7\text{N}$

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## Parity

It was introduced by Eugene Wigner in 1927 to describe the symmetry of the atomic states with respect to the „**space reflection**“. (When  $\vec{r} \rightarrow -\vec{r}$  changes direction)



E. Wigner (1902-1995)

### Symmetries and conservation laws

Emmy Noether (1882-1935) mathematician discovered in 1915 that the conservation laws are closely related to the symmetries of the equations.



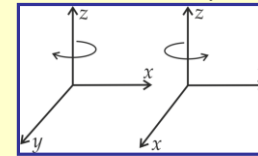
E. Noether (1882-1935)

**Symmetry:** invariance against certain transformations

For example:

- Symmetry for **time shift** (past-present-future)  $\rightarrow$  **energy** conservation
- Symmetry for **spatial shift** (here or there)  $\rightarrow$  **momentum** conservation
- Symmetry for **orientation** (any direction)  $\rightarrow$  **angular momentum** conservation
- **Gauge** symmetry of Maxwell-equations  $\rightarrow$  **electrical charge** conservation
- ... etc.

Laws of physics should not depend on whether we use a right-handed or left-handed coordinate system!



Left-handed right-handed

Can be shown that

$$\vec{r} \Rightarrow -\vec{r}$$

Moving from one to the other is equivalent to mirroring!

cannot be made overlap by any rotation!

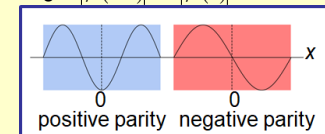
**Parity:** symmetry against central mirroring

The probability distribution should not change:  $|\psi(-\vec{r})|^2 = |\psi(\vec{r})|^2$

This can be done in two ways:

Positive parity:  $\psi(-\vec{r}) = +\psi(\vec{r})$

Negative parity:  $\psi(-\vec{r}) = -\psi(\vec{r})$



The parity of the microphysical states are either +, or -, well defined!!!

## Magnetic dipole moment

The magnetic dipole moment is connected to the angular momentum!

$$\vec{\mu} = g \cdot \vec{J} \cdot \mu_N, \text{ where } \mu_N = \frac{e \cdot \hbar}{2M_p \cdot c} = 5,050 \cdot 10^{-27} \left[ \frac{\text{J}}{\text{T}} \right] = 3,152 \cdot 10^{-8} \left[ \frac{\text{eV}}{\text{T}} \right]$$

gyromagnetic coefficient      nuclear magneton

**Consequences:**

- Even-even nuclei:  $\vec{\mu} = 0$
  - Even-odd nuclei:  $\vec{\mu} = \vec{\mu}_{\text{odd}}$  (the magnetic moment from the unpaired nucleon)
- There can be two causes for magnetic moment („two types“ of angular momenta):

- Orbital movement  $\vec{\mu}_L = g_L \cdot \vec{L} \cdot \mu_N$

- Connected to spin:  $\vec{\mu}_S = g_S \cdot \vec{S} \cdot \mu_N$

	proton	neutron
$g_L$	1	0
$g_S$	2,793	-1,913

**Importance:** interacts with magnetic field!

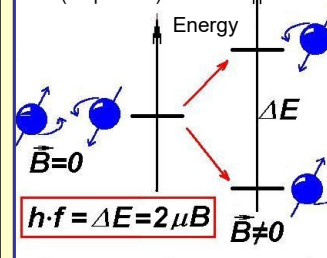
Interaction energy:  $E = -\vec{\mu} \cdot \vec{B} = -\mu_z \cdot B$ , if the z-axis is directed

in the direction of the  $B$  field.  $E = -\mu_z \cdot B = -g \cdot m_j \cdot \mu_N \cdot B$

## Practical use of nuclear magnetic moments: MRI

### NMR: Nuclear Magnetic Resonance

The basics of NMR (for protons)



(not only protons are suitable!)

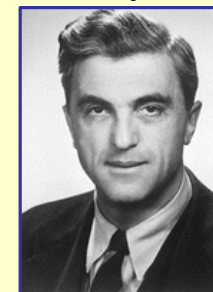
Based on two important parameters of nuclei:

- **angular momentum** (spin) (for protons:  $\frac{1}{2} \hbar$ )
- **magnetic dipole** moment

### MRI: Medical Resonance Imaging

1938-1945:

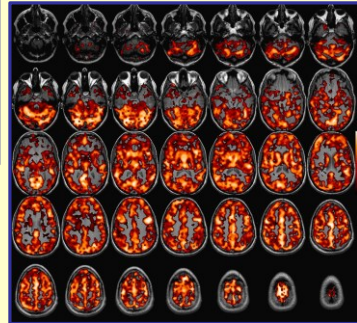
Felix Bloch and Edward Purcell have developed the NMR



Felix Bloch  
(1905-1983)



## The MRI-scanner



For example: brain tomography

## Excited states, energy levels

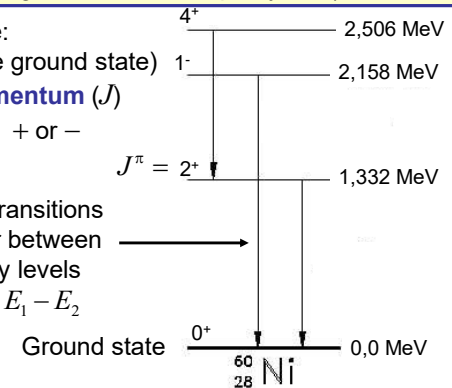
Nuclei are quantum mechanical systems: they have discrete energy states, with well defined quantum numbers (angular momentum, parity, etc.).

Properties of a state:

- **Energy** (above the ground state)
- Total **angular momentum** ( $J$ )
- **Parity** ( $\pi$ ) can be + or -
- **Half-life** ( $T$ )

Gamma-transitions can occur between the energy levels

$$h\nu = E_\gamma = E_1 - E_2$$



## Thank you for your attention!

These slides are **uploaded** in the „Files“ menu item of the Teams Group: *Nuclear and Reactor Physics Fundamentals*, in Channel: *Nuclear Physics 1. Basic Properties* (8. Oct.)

At the end of the slides there are some „Self-test questions“. Please try to answer them to check your own understanding.

## Self-test questions

1. Select out from the following nuclei  
 $^{45}_{20}\text{Ca}$ ,  $^{45}_{21}\text{Sc}$ ,  $^{40}_{20}\text{Ca}$ ,  $^{45}_{22}\text{Ti}$ ,  $^{45}_{23}\text{V}$ ,  $^{44}_{20}\text{Ca}$ ,  $^{44}_{24}\text{Cr}$ 
  - the isobars
  - the isotopes
2. Why should the apparatus be in vacuum at the Rutherford-experiment?
3. Why is different the „Coulomb-hill“ inside the positively charged part from the outside part at the Rutherford experiment?
4. Hofstdter needed electrons of 300 MeV to determine the size of the nucleus. Why were ~ 5 MeV alpha-particles enough for Rutherford?

### Self-test questions (cont.)

5. Why is a velocity selector needed in Aston's mass-spectrograph before the analysing magnetic deflection?
6. Where would the naphthalene and nonane ions hit in a mass-spectrograph, if there was no binding energy in the nuclei, and if the mass of the neutron and proton was the same?
7. What is the difference between energy and binding energy?
8. What are the main assumptions of the liquid drop model? What do we learn about the interaction between the nucleons from this model?
9. A liquid drop is held stronger together because of the surface tension of the liquid. Why do we say then, that the surface energy weakens the binding of the nuclei?
10. For  $A=\text{const.}$  the average energy of a nucleon ( $\varepsilon$ ) is described by 3 parameters of a parabola:  $a$ ,  $Z_{\min}$ , and  $\varepsilon_{\min}$ . Derive the  $A$ -dependence of these parameters from the Weizsäcker-formula!

### Self-test questions (cont.)

11. Why is the average energy of a nucleon ( $\varepsilon$ ) an important parameter?
12. Which nuclear energy level is described by the Weizsäcker formula?
13. How would look the  $Z_{\min}(A)$  and the  $\varepsilon_{\min}(A)$  functions, if there was no Coulomb-repulsion? (all other terms would remain in the Weizsäcker formula)
14. What parameters are usually used to characterise a nuclear energy level?
15. What are the possible values of the parity of a level?
16. Does the „pairing energy“ in the Weizsäcker formula has something to do with the parity? Clarify both!
17. What kind of radiation is emitted when a nucleus decays from an excited state to a lower lying state?