Energy and mass of atomic nuclei

The energy and mass of atomic nuclei are closely related, based on Einstein's well-known formula: $E = mc^2$. Here c is the speed of light in vacuum. Energy relations are important in the decay of atomic nuclei, since the decay products acquire some kinetic energy during decay, and this can only be achieved if the energy (mass) of the initial state is slightly greater than the energy (mass) of the final state.

Energy-conservation (during decay)

The energy release in a decay is:

$$Q = E_{\text{initial}} - E_{\text{final}} = \left(m_{\text{initial}} - m_{\text{final}}\right) \cdot c^2 . \qquad (1)$$

The name of Q is: decay energy. Of course, the masses in this formula represent the total mass of all the particles involved in the decay. That is, for example, in alpha decay, when a daughter nucleus and an alpha particle are formed in the final state, then $m_{\text{final}} = m_{\text{daughter}} + m_{\text{alpha}}$.

Determination of nuclear masses

The mass (and energy) of the currently known more than 2,000 (stable and radioactive) nuclei have been determined in several ways. There are nuclei whose mass can be measured by some direct method, such as mass spectroscopy. The mass of many other nuclei can be determined indirectly from the measurement of decay or nuclear reaction energies with the help of "known" nuclear masses. There are artificially produced nuclei, usually with a very short half-life, whose mass cannot be precisely determined even in this way, but can be estimated based on some nuclear model or some other systematic behaviour. The very diverse nuclear reactions of more than 2,000 nuclei and the many different measurements performed on them provided an amazing amount of data of varying accuracy. Reviewing these, checking their relationship, correlation, consistency or contradiction, and extracting the most probable values from these is a task that has occupied special groups of researchers for years. As a result of these researches, tables are created that contain the most consistent values that can be read from the experimentally available data. The present program is based on *G. Audi and A.H. Wapstra*: Experimental Atomic Masses, Nucl. Phys. A595 (1995) p409.

Nucleon number (mass number) conservation

Several physical quantities are "conserved" in nuclear transformations. We already mentioned conservation of energy (1). Another important conservation law is nucleon number[1] (mass-number) conservation. This means that at the end of any transformation there must be exactly the same number of nucleons in the final state as there were in the initial state.

$$A_{\text{final}} = A_{\text{initial}} \qquad (2)$$

Of course, in the final state the number of all nucleons must be added together as well. That is, for example, during alpha decay in the final state: $A_{\text{final}} = A_{\text{daughter}} + A_{\text{alpha}}$.

[1] The atomic nucleus is composed of protons and neutrons. These are collectively called nucleons. The nucleon number – also called mass-number – is therefore the combined number of protons and neutrons. It is traditionally denoted with the letter A.

Binding energy and missing mass

At first glance, there seems to be a contradiction here. If there is always the same number of nucleons (e.g. A) in the initial and final states, and the mass of a nucleon is m_{nucleon}, then the mass of the initial and final states should be the same! $m_{\text{initial}} = A \cdot m_{\text{nucleon}} = m_{\text{final}} = A \cdot m_{\text{nucleon}}$. However, based on (1), it should follow that $Q = 0$, i.e. there would be no energy for any atomic nucleus to decay!

The fact is that the nucleons that make up the atomic nucleus are not only closely packed particles (like the bricks of a house), but they are **bound** by a strong attractive interaction! Therefore, the mass of an atomic nucleus is usually smaller than the sum of the masses of the nucleons that make it up!

$$m_{\text{nucleus}} = Z \cdot m_{\text{proton}} + (A - Z) \cdot m_{\text{neutron}} - \Delta m \qquad (3)$$

In this formula, Z is the number of protons and A is the number of nucleons (mass-number). This formula also takes into account that the masses of the two nucleons - the proton and the neutron - are slightly different. The name of Δm in the formula is: **missing mass**. This is characteristic of how strongly these particles are bound inside the nucleus. The $B = \Delta m \cdot c^2$ quantity is called the **binding energy** of the nucleus. We have to invest this energy if we want to split the nucleus into its constituent protons and neutrons. Consequently, the mass of the nucleus is smaller than the sum of the masses of its constituent parts!

Data displayed by the program

Selecting the nucleus

The composition of an atomic nucleus can be determined with TWO data: the nucleon number (or mass-number) A and the number of protons Z must be given. Obviously, the number of neutrons: $N = A - Z$. The proton number can be specified directly, or it can also be specified by the chemical symbol of the atom, since this unambiguously determines the number of protons. That is, there are three possible "input" fields in the program: A, Z and chemical symbol. We can start the selection of the nucleus with any of them. When one of these is entered (and the data entry is closed with the ENTER key), the program lists the nuclei found in its database that meet the given condition. From this list, we can then select the nucleus whose energy relations we want to display.

Display of energy conditions

The energy conditions are shown numerically in a table on the left side of the screen, and graphically on the right side of the screen.

All energy values are given in kiloelectronvolt (keV) units. 1 keV = $1.60217733 \cdot 10^{-16}$ J. In microphysics, the energy units used are the electron volt (eV) and its multiples. A further simplification is the introduction of the "atomic mass unit". The symbol for the atomic mass unit is u, (formerly amu = atomic mass unit), and its definition is: one twelfth of the mass of a carbon atom with mass number 12. That is

$$1\ \text{u} = \frac{m\left({}^{12}\text{C}_{\text{atom}}\right)}{12} = 931493.614838934\ \text{keV/c}^2 \qquad (4)$$

Please note how accurately this quantity is known (measured).

- **Mass excess (keV).** Since nucleon number (mass number) is conserved in nuclear processes, the $A \cdot u \cdot c^2$ quantity is constant during nuclear processes. In order to avoid calculations with too high energy values (and numerical errors occurring when subtracting large numbers), it is advisable to enter only the deviation from this value for each nucleus. Therefore, the value given in the table:

$$E_{\mathrm{x}} = m_{\mathrm{ex}} \cdot c^2 = m_{\mathrm{nucleus}} \cdot c^2 - A \cdot u \cdot c^2 \qquad (5)$$

Please note that this is not the same as the missing mass, which was included in the expression of the binding energy, since here ALL nucleons are taken into account with the same mass: u! Accordingly, if we are interested in the total energy (mass) of a nucleus, we can obtain it as follows:

$$m_{\mathrm{nucleus}} \cdot c^2 = A \cdot u \cdot c^2 + E_{\mathrm{x}} \qquad (6)$$

That's why - to distinguish - we call this **mass excess**. Of course, its value can be positive or negative (or even zero).

- ***Binding energy (keV)*** We have already touched on this concept in the previous sections. It can be calculated as: $B = \left[Z \cdot m_{\mathrm{proton}} + (A - Z) \cdot m_{\mathrm{neutron}} - m_{\mathrm{nucleus}} \right] \cdot c^2$. Since the atomic nucleus is always a bound system (if it were not, the nucleons would not stay together), the binding energy is always positive, i.e. $B > 0$.
- ***Energy/nucleon (keV)*** According to the $E = mc^2$ formula, the energy is zero if $m = 0$. In other words, the starting point (origin) of the energy axis is at "nothing" (empty space). In this system, the energy of all existing objects is always positive. However, when examining bound systems, it is usual to place the zero point of the energy axis when the parts that make up the system already exist, but are not yet in a bound state (e.g. they are very far from each other). In this case, we take the energy of the (already created but) not-yet-bound system as zero. The energy of the bound system should be smaller than this – i.e. negative. It is easy to see that in such a choice of the coordinate system the following relationship exists between the energy of the system and the binding energy: $\tilde{E} = -B$, since it is precisely the binding energy that must be invested in order to bring the system to a non-bound state, i.e. zero energy $\left(\tilde{E} + B = 0 \right)$. The energy/nucleon shows how "deeply" a single nucleon is bound on average in the attractive potential that holds the nucleus together.

  Based on these, the displayed **Energy/nucleon** $= \dfrac{\tilde{E}}{A} = -\dfrac{B}{A}$.
- <u>**Q-beta, Q-EC, Q-alpha**</u> Based on formula (1), these values give the decay energy of negative beta decay, electron capture, and alpha decay, respectively. Of course, if we know the masses, we can always calculate these values, which can be positive or negative. $Q > 0$ is a necessary (but not sufficient) condition for decay to occur. In the case of negative beta decay and electron capture (EC), the table shows values only if they actually occur. In such cases, however, the experimentally measured values are usually included in the table, not those calculated from the mass differences! It may happen that the masses of the initial and final atomic nuclei are much less precisely known than their mass difference - which can be determined by the exact measurement of the decay energy.
- <u>**S-neutron, S-proton**</u> These are the separation energies of neutrons or protons. The separation energy shows how much energy would have to be invested to tear a neutron or a proton out of the nucleus. Note here that in several cases we also see $Q_{\mathrm{alpha}} < 0$ values in the table. Of course, in this case alpha decay cannot occur, but we can still give an interpretation to the indicated value. The value shows how much energy would have to be invested to tear an alpha particle from the nucleus. So this can be interpreted as the separation energy of the alpha particle, i.e. $S\_\mathrm{alpha} = -Q_{\mathrm{alpha}}$, if $Q_{\mathrm{alpha}} < 0$

In addition to the values shown in the table, we also indicated – based on the above-mentioned data-base – the uncertainties that could be derived from the measurements. These values are listed in the +/– column. However, there are cases when direct measurement results were not available (when the database was created), but an estimated value could be given based on either known mass data or nuclear models. In this case, there is a SY symbol in the **Note** column.

Graphic display

On the right hand side of the screen the energy conditions are also displayed graphically in accordance with the numerical data. On the vertical axis are the energy differences compared to the ground state energy of the studied nucleus. As additional information, the graphic display also shows the daughter nuclei created after individual decays and separations. Each decay is marked with a different color: negative beta decay is indicated by green, electron capture (EC) is indicated by purple, and alpha decay is indicated by blue. The starting nucleus is always red.

The **positive beta decay** is not shown separately either in the table or in the program. Positive beta decay can occur if the energy available for electron capture (EC) is $Q\_\mathrm{EC} > 2m_e c^2 = 1022\ \mathrm{keV}$. Here m_e is the mass of the electron, and c – as usual – the speed of light in vacuum. After positive beta decay, the composition of the daughter nucleus is the same as after electron capture (EC). Therefore, there is no need for a separate display.