Simulation of cyclotron operation

(C. Sükösd 2014)

Introduction

On 5th January 1929 Leo Szilárd Hungarian physicist applied for a German patent relating to a particle accelerator in which the particles move in a circular path. Independently, on 6th May 1929 the Hungarian Sandor Gaál sent also an article with similar content to the Zeitschrift für Physik German scientific journal. His paper was not published because the editors misunderstood the content, and thought that he reinvented the accelerator concept of Wideröe. Also independently of them, in 1932 American physicist Ernest Orlando Lawrence built the first working cyclotron, for which he received the Nobel Prize in 1939.

In the cyclotron a homogeneous magnetic field keeps the particles on circular tracks, and they usually perform circular motion with constant speed (the absolute value of their velocity is constant). However, when they pass through the "accelerating gap" – which is along a diameter of the circle – even an electric field acts on them along their track (in addition to the constant magnetic field), which makes the absolute value of their speed - and also their energy - increase. Since the orbiting particles pass through the accelerating gap from time to time again, they can be accelerated with the same voltage again and again many times, provided that the voltage in the acceleration gap varies "in sync" with the circulating particles.
This is the basic principle of the machine that is now known as cyclotron.

The equations and relations necessary to adjust the cyclotron parameters can be found in the section "***Calculation hints***". The section "***Description of the Simulation***" program contains information on how to use the program

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Sample task

Prepare a proton beam of approx. 20 MeVenergy with the cyclotron of 200 cm radius.

The better will be the result,
• ... the more accurately the average beam energy approaches the 20 MeV;
• ... the smaller is the standard deviation of the beam energy;
• ... the greater proportion of the particles hit the target;
• ... the smaller will be the radioactivity of the limiter and the cyclotron wall.

Description of the Simulation

The cyclotron in the simulation is a "fictitious" device; no real machine exists with these parameters.
The left side of the screen contains the adjustable operating parameters. In the upper part of the right hand side a schematic diagram of a cyclotron is shown. In "reality" the cyclotron radius is 200 cm. In the bottom part of the right hand side a few graphs can be selected. Their role is to show the results.

First we list the input fields of the left hand side of the screen, where individual operating parameters can be set.

Magnetic field

In the picture of the cyclotron the black the area is the region where a permanent magnetic field (perpendicular to the screen) acts on the particles. The maximum magnetic field is 0.5 T.

The light blue rectangle in the upper part of the picture is a domain (also inside the cyclotron vacuum chamber) where the magnetic field is 0, which causes the particles move in a straight line from "left to right".

Accelerating voltage

The vertical white line on the diagram represents the accelerating gap of the cyclotron. Here, an electric field acts on the particles, and can accelerate them. On both sides of the accelerating gap are the two electrodes (the professionals call them "D" - on the basis of their shape). They act as “Faraday cap”, so inside them no electric field acts on the particles, electric field is only between them, in the accelerating gap. The accelerating field is produced by switching alternating voltage on the two D-shaped electrodes. The accelerating field in the picture is always in "horizontal" direction.

The ***frequency*** and the ***amplitude*** of the accelerating voltage can be set in the corresponding input fields. The actual value of the AC voltage is indicated in the right-hand side of the picture with the dot periodically moving to left and right. (Think of it as if the left D was grounded, and the moving dot would indicate the current value of the right D compared to this potential.) Note that no arbitrary frequency can be chosen, as the two electrodes form a cavity, which can "resonate" only in certain frequency range. The permissible frequency range is from 3 MHz to 10 MHz. Similarly, the oscillator producing the accelerating voltage can produce amplitudes only up to 1000 kV (in reality these values are different; we take these values here to make the simulation task easier). For the sake of orientation the energy change (dE) of the most energetic particle is also displayed under the picture of the cyclotron.

Particles

In this input area one can select the type of the particles to be accelerated. The cyclotron can handle three types of particles: protons, deuterons and alpha particles (He^{2+} ions).

Simulation

Three parameters can be set here. We can ask that the program starts only ***one single particle*** (this is useful during the “set-up” time), but it is possible, that the program launches particles ***continuously*** at randomly chosen time (this should be selected when we want to create a “beam” of particles).

The “***Slow down***” input parameter influences only the display, not the calculation. If we want to slow down the simulation – for example to observe the movement of the particle more carefully – a larger number should be set. When we reach the speed capability of the processor, the simulation will not speed up anymore, even if we set her an even smaller value.

If we check the Trajectory checkbox, the program draws the trajectory of the particles. If many particles are accelerated the same time, the program draws only part of the trajectories for most of the particles – to keep the screen clear.

Particles at the target:

It is important for the users to get a beam where the energy of the particles does not deviate too much from the specified value. The best would be if each particle had exactly the same energy. These fields indicate how much this goal has been achieved. You can see the mean value of the energy of the incoming particles, and the standard deviation. These values are set by the program; they cannot be set by the user.

Ion source, limiter, cycklotron wall, radioactivity

Only charged particles can be accelerated, therefore the electrically neutral atoms must be ionized. This task is performed by the ***ion source***. The place of the ion source is shown by a small white square around the centre of the cyclotron. The limiter is a metallic piece, which absorbs the particles that hit it, and this way it prevents them to move further. The place of the limiter is shown by a small blue rectangle in the picture. In the ***Ion source*** and ***Limiter*** input fields their Y coordinate can be set. Their X coordinate is always 0. The origin is in the centre of the cyclotron. Because of mechanical reasons the centre of the ion source can only be place in the a [0,–40] cm interval. The height (Y-dimension) of the limiter can also be set.
The metallic ***cyclotron wall*** (also blue in the picture) also absorbs the particles that hit it, like the limiter does. It is important to note that every particle which has energy greater than 0,45 MeV creates ***radioactivity*** via nuclear reactions, if they hit either the limiter or the cyclotron wall. The created radioactivity is a monotone increasing (but not linear) function of the energy/mass of the particle. The radioactivity also decreases according to the exponential decay law. This way sooner or later we arrive to a radioactive equilibrium, where around the mean value some fluctuations can be observed. The quality of the beam production is also depending on the created radioactivity. For a given beam, the quality is better if less radioactivity was created while preparing the beam.

Results

The achieved simulation results can be evaluated by inspecting the graphs of the lower right-hand part of the screen. Three graphs can be selected: ***Energy distribution***, ***Time distribution*** and ***Further data***. These tests can only be meaningful when the simulation was run in ***continuous*** mode, and particles have already arrived to the target. In the graph of the ***Further data*** the following information are plotted: the number of particles that hit the ***target***, the ***limiter*** and the ***cyclotron wall***, but here we also see how radioactive the limiter and the cyclotron wall has become because of the particles that hit them. (The numbers showing the level of radioactivity are in arbitrary units). Of course, the useful particles are in the "***Target***" column.
The ***Energy distribution*** and the ***Time distribution*** graphs are prepared using only the "useful" particles that arrived on the target. One of them shows the distribution of the particle energy (MeV), the other one shows the distribution of the time (in seconds). (Note that the extracted cyclotron beam is not continuous in time, but particles arrive to the target only around certain points of time – in sort of “packets”.)
The scale of the vertical axis can be changed by the up/down arrow keys next to the axis.

Calculation hints

The following equations and relationships can help in setting the cyclotron parameters.

1) A force acts upon a particle with q charge, which moves with velocity in a $\boldsymbol{B}$ magnetic field, which is perpendicular to its velocity vector. The absolute value of this force is: $F = qvB$. The direction of the force is perpendicular on both the $\boldsymbol{B}$ vector and on the v velocity vector. This force is called Lorentz force, and this represents the centripetal force necessary to circular motion: $mv^2/R=qvB$. Here m is the mass of the particle; R is the radius of the circle. Therefore, the angular frequency of a charged particle moving in a circle in homogeneous magnetic field can be written as: $\omega = v/r = qB/m$.
The frequency: $f = \omega/2\pi = qB/(2\pi m)$. If the charge of the particle is $q = Z\cdot e$, and its mass is $m = A\cdot m_u$, then $f = e/(2\pi m_u)\cdot Z\cdot B/A=15{,}3575\cdot(Z\cdot B/A)$ [MHz].

The absolute value of the velocity: $v = (qB/m)R$, and therefore the kinetic energy: $E=(mv^2)/2=(q^2B^2/2m)R^2$.

2) If a charged particle (charge = q) crosses U electrostatic potential difference, its energy changes. The change in the energy is: $\Delta E = qU$. This way, also the energy of the particle that traverse the accelerating gap of the cyclotron is changing. Since the actual value of the potential difference is not constant (alternating voltage), naturally one should use the actual value of $U(t)$ in the moment when the particle crosses the accelerating gap!

3) In the simulation the time dependence of the accelerating voltage is the following:

$$U(t)=U_0 \cos(2\pi f t)$$

Here f is the frequency, U_0 is the amplitude. Note, that the accelerating voltage is maximal in the “starting” moment $(t=0)$.