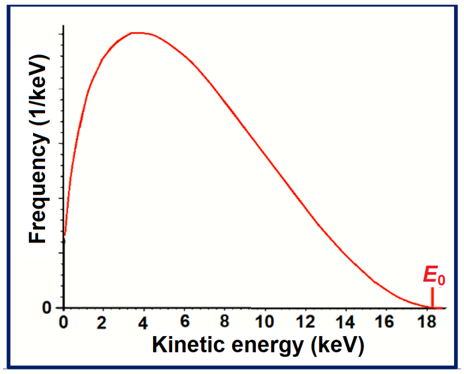
National Leo Szilárd Physics Competition 2024
Computer simulation task
Theoretical introduction

Measurement of the spectrum of beta-radiation with Wien filter

It is known that during the beta decay of atomic nuclei, the emitted electron and antineutrino "share" the total energy released during the decay. For this reason, the energy of the emitted electrons can take any value from zero to a maximum energy (E_0). Due to the method of this "sharing", electrons of different energies come with different frequencies.

Note: Also the nucleus takes part in this „sharing" and gets some momentum, but its effect on he energy is negligible, because the nuclear mass is much larger than that of the other two particles.



The **energy spectrum** of electrons shows the frequency distribution of the electrons in function of their kinetic energy. Such a distribution is presented in the figure above. In this simulation, the task will be to measure such a spectrum and determine the maximum energy (E_0) as accurately as possible.

The simplest, of course, is if we have a detector that would immediately measure the energy of the electrons and could produce a histogram based on the numbers of electrons of different energies during the measurement time. Such spectroscopes (e.g. based on semiconductor detectors) do exist. In this simulation, however, we are trying to determine the energy spectrum of the radioactive sample with a really simple device, based on electromagnetic deflection of the particles – the Wien filter.

The spectrum and its measurement

Before we get down to the specific task, it's worth better understanding the meaning of the spectrum! When we measure the distribution of particles according to energy, the hit number of our detector is determined by the quantity $\Delta N = N_0 \cdot f(E) \cdot \Delta E$, where N_0 is the number of all particles that entered the measuring device during the measurement time, $f(E)$ is the shape of the spectrum, and ΔE is the "energy window" into which the particles fall that are "selected" by our measuring device. . If the energy window is very narrow, then it has to be measured for a long time, since only very few particles enter it (ΔN will be small). With a larger ΔE one, however, the energy resolution of our equipment will be worse. Therefore, you usually have to make a reasonable compromise when choosing ΔE . Based on this, the spectrum can be measured based on the following simple relationship:

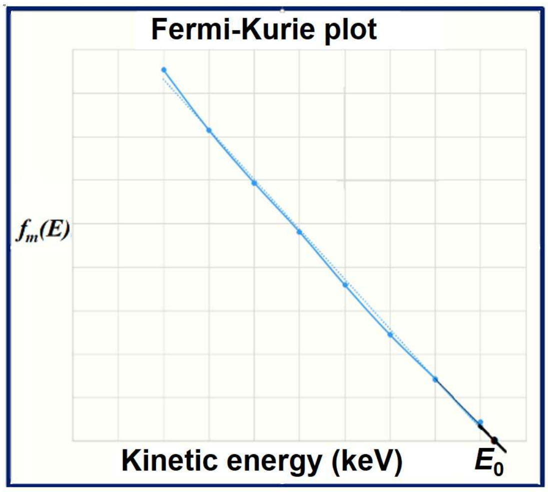
$$f(E) = \frac{1}{N_0} \cdot \frac{\Delta N}{\Delta E} \qquad (1)$$

This way we also understand why [1/energy] is the unit for the vertical axis of the above spectrum. The important experience that can be drawn from this connection is that it is not enough to measure the ΔN number of hits, but also the ΔE size of the "energy window" must be known!

The "end" of the spectrum, determination of the E_0 decay energy

It can be seen from the shape of the example spectrum above that the exact experimental determination of the decay energy is not easy, because the function "smoothes" into the horizontal axis around the end, and this makes difficult the exact determination of the ending point. This difficulty is solved by the Fermi-Kurie plot, which was introduced by E. Fermi Italian, and F.N.D. Kurie American physicists. This plot is based on the "linearization" of the beta spectrum, which can be done using the theory of beta decay developed by E. Fermi. (If you are interested in the specific implementation of this, you can find it in the Appendix.)

Therefore $f_m(E) = \text{konst} \cdot (E_0 - E)$. The advantage of this transformation is that the linearized $f_m(E)$ is a linear function of the energy (at least for the so-called "allowed" type beta decays, around the E_0 decay energy). Therefore, when plotting this function, the straight line intersects the horizontal axis exactly at E_0, and thus enables a much more precise determination of the E_0 decay energy (see figure). During the competition, a pre-programmed EXCEL table will help to create the $f_m(E)$ data series (the linearization) from the measured data $f(E)$.



The operation of the Wien-filter

The Wien filter applies mutually perpendicular, homogeneous electric and magnetic fields. In the optimal case, these fields are even perpendicular to the velocity of the incoming charged particles (electrons in our case). Therefore, an $(q\vec{\mathcal{E}})$ electrostatic force and a $(q[\vec{\mathbf{v}}\times\vec{\mathbf{B}}])$ magnetic Lorentz force act on the particles. Here, q is the charge of the particle (negative elementary charge in the case of electrons), $\vec{\mathbf{v}}$ is the speed of the particles, $\vec{\mathcal{E}}$ is the electric field strength, and $\vec{\mathbf{B}}$ is the magnetic field. These last three are vector quantities. (The $\vec{\mathcal{E}}$ notation is used for electric field strength to distinguish it from the E energy.)
With appropriate choice of the electric and magnetic field strengths one can achieve

$$q\vec{\mathcal{E}} - q\left[\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right] = 0,$$

i.e the resulting force acting on the particle will be zero. The condition for this (in addition to the orthogonality of the field strengths and the velocity) is the following:

$$v = \frac{\mathcal{E}}{B} .$$

(Here, the quantities on the right are the absolute values of the field strengths.)
This means that such a device can sort and "filter" incoming particles based on their speed. If, therefore, many particles of different velocities enter into the device through a very small hole, and exit the Wien-filter and enter into the detector through a very small hole exactly opposite the entrance, then only those with exactly the above speed will be detected, since only they will move in a straight line. Particles with different speeds will be deflected and they will hit somewhere in the wall of the device. Of course, in order to determine the velocities with infinite precision, the entry and exit slits should be infinitesimally small - which is not feasible from a practical point of view. With slits of finite size, there will always be a small $(v, v+\Delta v)$ velocity-interval in which particles fall through the filter. This interval will also determine the ΔE "energy window" of the filter.

Appendix

How to "linearize" the spectrum:

Let us form the following expression from the "measured" $f(E)$ spectrum:

$$f_m(E)=\left[\frac{f(E)}{F(Z,E)\cdot\left(E+mc^2\right)\cdot\sqrt{\left(E+mc^2\right)^2-\left(mc^2\right)^2}}\right]^{\frac{1}{2}}.$$

Here m is the rest-mass of the electron, Z is the atomic number of the daughter nucleus (after the decay) and $F(Z,E)$ is the so-called Fermi-function.

The Fermi-function: $F(Z,E)\cong\dfrac{2\pi\eta}{1-e^{-2\pi\eta}}$, where $\eta=Z\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\hbar c}\right)\cdot\dfrac{E+mc^2}{\sqrt{E(E+2mc^2)}}$.

Based on Fermi's theory, it can be proven that for the so-called "allowed" type beta decays $f_m(E)=\mathrm{konst}\cdot\left(E_0-E\right)$ in quite a large region around the decay energy.

Note that the dimensionless quantity in parentheses in the formula of η is the fine structure constant usually denoted by α, which is characteristic of the "strength" of the electromagnetic interaction:

$$\alpha=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}\approx\frac{1}{137}.$$

If we also notice that in the above formula $\dfrac{E+mc^2}{\sqrt{E(E+2mc^2)}}=\dfrac{c}{v}\equiv\dfrac{1}{\beta}$, where v is the speed of the electron, then the formula can easily be memorized: $\eta=Z\dfrac{\alpha}{\beta}\approx\dfrac{Z}{137\beta}$.