SIMULATION OF ATTENUATION OF A RADIATION BEAM

The shielding against radioactive radiation is an important task of the radiation protection. The knowledge of the attenuation of radiation helps in the accomplishment of this task.

The type of the radioactive radiations can be different (for example alpha-, beta-, gamma-radiation or neutron-radiation). They also interact differently with matter; therefore their attenuation follows also different laws. This program simulates the attenuation of a beam of particles that interact with the matter with **low probability**, and after the first interaction the particles will **disappear from the incident beam**. There can be two causes of the “disappearing”: the particles get either **absorbed**, or **scattered out** of the beam. In the following when we talk about disappearing, we always refer to one of these causes.

The photons in the gamma-radiation are such kind of particles, and the neutron radiation behaves approximately this way as well. At the end of this document we will say a few words also about the absorption of the alpha-radiation, which follows different laws than those we deal with in this simulation.

As mentioned above, the program simulates attenuation of radiation, where the particles (for example gamma-photons) can fly by many atoms until they finally interact with one atom and disappear from the beam. This behaviour can be described by the concept of the **interaction probability per unit path length**.

> ***Note***: More precisely, this is a probability density; its measure is 1/distance (for example 1/cm). There are different names in use for the same quantity in different fields of applications. In some fields it will be called as **linear attenuation coefficient**, or **total macroscopic cross section**. (However, this latter name is a bit misleading, since this is not an area-like quantity: its measure is 1/cm and not cm^2).

Theoretical description

Suppose that N_0 particles enter into a bulk of material (during a certain time). Denote the number of particles reaching the depth *x* by $N(x)$ (see Fig. 1.).

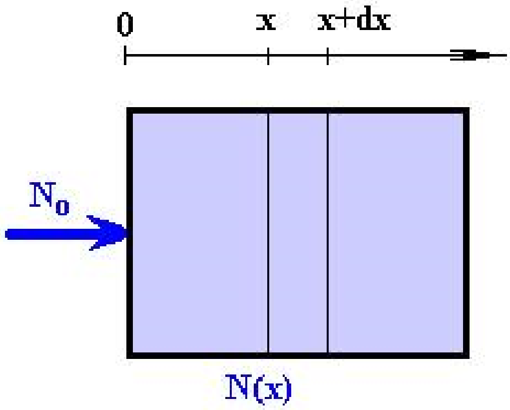

Fig. 1.

Denote the interaction probability per unit length by μ. Then the probability that a particle disappears from the beam in Δx distance is: $\mu \cdot \Delta x$. The expected number of the disappearing particles from the

$N(x)$ particles in the $(x, x+\Delta x)$ interval is: $N(x)\cdot\mu\cdot\Delta x$. So many particles will be missing from the beam, therefore the change (decrease) of the number of particles: $\Delta N = -N(x)\cdot\mu\cdot\Delta x$. Rewriting this equation we get: $\frac{\Delta N}{N} = -\mu\cdot\Delta x$.

Although the change in the number of particles can only be integer (since 0.01 particle cannot disappear), but having a very large number of particles $(N \to \infty)$ we can make a limes in both sides of the equation, and get a differential equation: $\frac{dN}{N} = -\mu\cdot dx$.

Let us integrate this differential equation! While *x'* goes from 0 to *x* on the right-hand side, *N'* will go from N_0 to *N(x)* on the left-hand side: $\int_{N_0}^{N(x)}\frac{dN'}{N'} = -\mu\int_0^x dx'$. We get: $\ln N(x) - \ln N_0 = -\mu\cdot(x-0)$, which can be rewritten as:

$$\boxed{N(x) = N_0\cdot e^{-\mu\cdot x}} \qquad (1)$$

This is the **exponential attenuation law**. Since we made an $(N \to \infty)$ limes while deriving this law, this is exact only for a beam consisting of infinitely large number of particles. For a beam consisting of finite number of particles there can be fluctuations from this expectation value because of the statistical nature of the phenomenon.

Please note that for this type of radiation one cannot speak about the „range", since there is no such a distance where no particle can penetrate. For this type of radiation the **half-value layer** is the appropriate quantity that describes the attenuation.

Half-value layer (or half-value thickness) is the thickness of the material at which the intensity of radiation entering it is reduced by one half. Let us denote it by L. According to the equation above:

$N(L) = \frac{N_0}{2} = N_0\cdot e^{-\mu\cdot L}$, from where we get:

$$\boxed{L = \frac{\ln 2}{\mu} \approx \frac{0{,}7}{\mu}}. \qquad (2)$$

The simulation

This program simulates the attenuation of a beam of particles that interact with the matter only with **low probability**, and after the first interaction the particles **disappear from the incident beam** (get absorbed or get scattered out). The simulation draws random numbers for every "step" (1 cm length) of every particle and determines if it will disappear or not in the next step.

In the upper-right field of the screen the particle beam arrives from the left, and falls on a grey-coloured block of material (100 cm thick), where the particles may interact with the matter with the interaction probability density given in the left part of the screen. The graph in the right-bottom shows the number of particles reaching different depths of the material. After a sufficiently large number of particles the exponential trend of the attenuation can be well observed. The observed fluctuations are caused by the statistical nature of the phenomenon.

Advice: The easiest way to observe the relationship between the half-value layer and the interaction probability is choosing μ = 0,007. Then according to the (2) equation we get *L*~100 cm for the half-value thickness.

ADDITIONAL NOTES

Gamma-radiation

Since the gamma-radiation is one of the types of the radiations that follow the exponential attenuation law, therefore we deal with it a bit more detailed.

The high-energy gamma-photon usually propagates „undisturbed" through the material, until it interacts with the matter with certain probability.

Three types of interaction are possible: photo-effect, Compton-scattering and pair-production.

Photo-effect occurs when the gamma-photon gives away its full $h\nu$ energy to a (strongly bound) electron and the electron gets kicked out of the atom. The kinetic energy of the kicked out electron is:

$$\frac{1}{2}mv^2 = h\nu - E_{binding} \tag{3}$$

In this interaction the total energy of the photon will be given away, therefore the photon disappears completely (not only from the beam).

Pair production occurs when the photon creates a positron-electron pair. In this process one part of the energy of the gamma-photon $\left(2mc^2\right)$ will be converted into the masses of the two particles. The created particles get their kinetic energy from the remaining part of the energy of the gamma-photon. Therefore the energy balance:

$$\left(\frac{1}{2}mv^2\right)_{electron} + \left(\frac{1}{2}mv^2\right)_{positron} = h\nu - 2mc^2 \tag{4}$$

In this interaction the total energy of the photon will also be given away, therefore the photon disappears completely. It is also clear that this process can only occur for photons whose energy is larger than the energy needed to create an electron-positron pair: $h\nu > 2mc^2$ (~1022 keV).

Compton-scattering occurs when the gamma-photon „scatters" on an electron. Here the gamma-photon does not disappear completely, but its direction (and energy) changes, therefore it scatters out of the original beam. The photon and the (free or slightly bound) electron „bounce" like two billiard balls. Let us denote the scattering angle of the photon by ϑ (the deviation from its original direction). The energy of the scattered photon is:

$$h\nu' = h\nu \frac{1}{1 + \frac{h\nu}{mc^2} \cdot (1 - \cos\vartheta)} \tag{5}$$

It is important to note that for every interactions above the photon disappears from the original beam: either it disappears completely (for the photo-effect and the pair-production) or it changes its direction of propagation.

Alpha-radiation, range

Unlike gamma radiation, the alpha radiation gets never strictly absorbed – it does not "disappear" - because the alpha particles are helium nuclei, and they remain there also after they stopped in the material. Since they are electrically charged particles they ionize the atoms and molecules as they fly nearby, therefore they also loose energy. As a result, they stop after a certain distance.
The distance at which the alpha particle is stopped is called the alpha-radiation **penetration depth** or the **range**. For a beam of alpha particles with the same energy each particle stops close to the range.
The penetration of the alpha particles in the material is shown in Fig. 2.

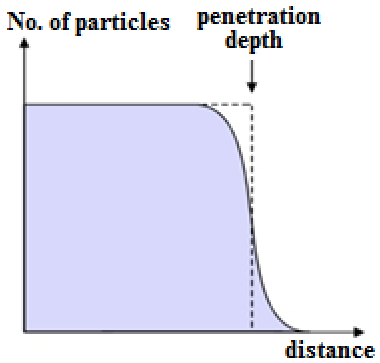

Fig 2.

It is important that the direction of the alpha-particle is not changing considerably during its travel in the material – because of its mass, which is relatively high as compared to the electron mass. Therefore the alpha-particle's path is more or less straight. A slight change in the direction at the end of the path causes the „blur" in the penetration depth in fig. 2.

As an example, Fig. 3 shows the paths of alpha-particles of 4,8 MeV energy in air (The figure was made by the TRIM simulation program.)

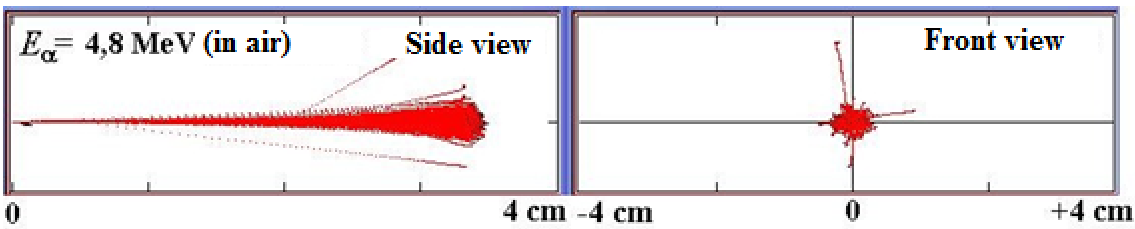

Fig. 3

Note: The energy-transfer $\left(\frac{dE}{dx}\right)$ along the path is described by the Bethe-Bloch formula. We cannot deal with it here. However, one can estimate the range – at least order of magnitude – without knowing the interaction mechanisms in detail. The order of magnitude of the energy of alpha-particles (of natural radioisotopes) is a few MeV (million electronvolt). On the other hand, the ionization energy of the atoms and molecules is around 0,1 – 1 eV (electronvolt). This means that the alpha-particle should ionize a few million atoms or molecules along its path until it loses its energy completely. The distance of atoms in a solid material is about 10^{-10} – 10^{-9} m, therefore the alpha-particle should travel around 10^{-5} – 10^{-4} m in order to fly by a few million atoms. Consequently, the range of the alpha-radiation in solid material is about 0,01 – 0,1 mm. The density of gases is about thousand times lower than the density of the solids; therefore the alpha-particle has to fly about thousand times longer distance to interact with the same number of atoms. Therefore the range in gases is around 1-10 cm, and obviously it also depends on the pressure of the gas.